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STATUS OF RANGE SAFETY FILTER SYSTEMS.(U)
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STATUS OF RANGE SAFETY FILTER SYSTEMS

RANGE SAFETY GROUP
RANGE COMMANDERS COUNCIL

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WHITE SANDS MISSILE RANGE
YUMA PROVING GROUND

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PACIFIC MISSILE TEST CENTER
ATLANTIC FLEET WEAPONS TRAINING FACILITY
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The goal of range safety is the prevention of injury to personnel or damage to property by taking all reasonable precautions consistent with operational requirements. This is dependent not only on precautions taken in the preparation of a missile or vehicle launch, but in the ability of the Range Safety Officer to maintain surveillance during flight to ensure compliance with established safety criteria. To maintain this necessary surveillance, the RSO must have at his disposal information depicting performance of the missile and possible impact locations.

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for comparison against predetermined destruct criteria as well as assurance that his entire safety system is in operational condition at all times. Presenting this information in a manner that allows clear and quick understanding of significant data will vary due to missile dynamics and test range geometry. Therefore, there is no one-best filter for all applications.

A survey of seven of the test ranges represented in the RSG of the RCC was made to determine the type of filter systems currently in use at the various ranges. This information is presented in this document as an aid to all ranges in determining which systems may have merit for their application and to provide some insight into future applications of filter systems.

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FOREWORD

The goal of Range Safety is the prevention of injury to personnel or damage to property by taking all reasonable precautions consistent with operational requirements. This is dependent not only on precautions taken in the preparation of a missile or vehicle launch, but in the ability of the Range Safety Officer (RSO) to maintain surveillance during flight to insure compliance with established safety criteria. To maintain this necessary surveillance, the RSO must have at his disposal information depicting performance of the missile and possible impact locations for comparison against predetermined destruct criteria as well as assurance that his entire safety system is in operational condition at all times. Presenting this information in a manner that allows clear and quick understanding of significant data will vary due to missile dynamics and test range geometry. Therefore, there is no one-best filter for all applications.

A survey of seven of the test ranges represented in the Range Safety Group (RSG) of the Range Commanders Council (RCC) was made to determine the type of filter systems currently in use at the various ranges. This information is presented in this document as an aid to all ranges in determining which systems may have merit for their application and to provide some insight into future applications of filter systems.

1.0 INTRODUCTION

1.1 Flight Safety Data Working Group

1.1.1 The formation of the Flight Safety Data Working Group (FSDWG) was originally considered at the 16th meeting of the Inter-Range Missile Flight Safety Group (IRMFSG). Member ranges were requested to select nominees for this working group. At the 17th meeting of the IRMFSG, held 28-29 April 1964, the FSDWG was established. The purpose of this working group was to assist the IRMFSG by studying, making recommendations to the IRMFSG chairman, and promoting the exchange of information between member ranges concerning generating, processing, and displaying flight safety data for the Flight Safety Officer (FSO).

The second and subsequent meetings of the FSDWG were held at the following ranges: Western Test Range, October 1964; NASA Houston, June 1965; Colorado Springs, March 1965; Eastern Test Range, June 1966; White Sands Missile Range, November 1966; Pacific Missile Range, May 1967; Eastern Test Range, May 1968; Wallops Station, October 1968; Air Force Satellite Control Facility, April 1969; Air Force Electronics Systems Division, October 1969.

1.1.2 From April 1964 through June 1966, the prime discussion of the FSDWG centered around the development of the various impact prediction systems being used at the RCC member ranges. At the November 1966 meeting (6th meeting), the group began a detailed investigation into the filtering techniques employed at these ranges. A detailed investigation into the Kalman technique was included. At the 9th meeting at Wallops Station in October 1968, it was agreed that each range would develop a subroutine, for exchange with other ranges, to be utilized for comparison of filter techniques.

This exchange occurred at the 10th meeting of the FSDWG and emphasis was then placed upon the development of a report of "Status of Range Safety Filter Systems," Task 6-25, issued at the 25th IRMFSG meeting of the IRMFSG, 25-26 September 1968.

1.1.3 An ad hoc committee was established to revise the original document and to add an ADTC section in March 1976.

1.2 Report

1.2.1 Task 6-25 was given to FSDWG in an effort to make a survey of real-time filtering and editing techniques presently in existence and publish the results in an IRMFSG document. This survey was to include system characteristics, advantages, disadvantages, etc.

1.2.2 At the FSDWG meeting held in October 1969, at the Air Force Electronic Systems Division, a proposed outline of the task was developed. This outline was arranged to provide four specific chapters covering the following areas: (1) a brief introduction into the background of the FSDWG and Task 6-25 (including the purpose of the report and the method of preparation); (2) a discussion of filtering as applied to flight safety, the instrumentation systems being utilized, and the computation and display systems available at the various ranges; (3) a brief and general outline of the types of filters being utilized with some descriptive information concerning their specific characteristics, and (4) the specific filters in use at the ranges surveyed, how and why they were developed, the mathematics and operation of the filters and their outstanding features and limitations.

1.2.3 Initial discussions for accomplishing this task indicated a need for general background information on

filtering systems and detailed information on specific filters in use at the RCC ranges. Each range was requested to supply detailed information on filter systems in use, why the system was developed, how it is used, theory of the system, outstanding features and limitations, and types of instrumentation data filtered.

2.0 DISCUSSION

2.1 The purpose of filtering data, for range safety systems, is to remove unwanted noise from the measured data in order to determine, as accurately as possible, the space position, velocity and acceleration of the target. There are, no doubt, as many filtering techniques being used as there are persons evaluating the results. However, with the proper computations, the results provide the FSO with information which can be readily used in determination of the actual trajectory. In order to accomplish this objective, data measurements from such instrumentation as radars are entered into a computer system, normally at high input rates such as 10-20 samples per second. These data must then be differentiated to obtain velocity and acceleration for use in calculation of predicted impact point and output to displays for use by the FSO. However, since all instrumentation systems produce erroneous data, errors must be removed before differentiation and prediction can occur. The computer system is also limited in speed and memory, therefore limiting the ability to statistically examine the data samples in real time These, in general, are the problems associated with data filtering for flight safety. The methods used at the various ranges surveyed are described more fully in Section 4.0 of this document.

2.2 Instrumentation Systems

2.2.1 A wide variety of instrumentation systems is used

to provide the FSO with information required to perform the decision-making process. This section is concerned with those instrumentation systems which provide data for input into the real-time computer. Historically, pulse radars have been the more popular of the real-time range safety instrumentation systems. These types of radars primarily measure the position vector only, i.e., range (R), azimuth (A) and elevation (E). Recently, doppler rate measurement devices have been added to some of these pulse radars to allow direct measurement of range rate. Most of these devices, with the exception of the FPS-16 rate kits, can operate with or without a coherent beacon; however, more accurate range rate measurement is obtained with a coherent beacon. Coherent beacons are in the development stage and their usage is expected to increase in the future.

Continuous wave (CW) systems have found usage where extremely accurate position and velocity measurements are required. These can serve the dual purposes of guidance system evaluation and range safety. For example, GERTS provides a direct measurement of three components of the velocity vector ($\dot{R}$, $\dot{P}$, and $\dot{Q}$). These systems are expensive to purchase and operate, and normally require expensive, special purpose transponders onboard the vehicle. There are other varied types of measurement systems in use at the ranges surveyed. Some of these measure position through interferometer techniques; others measure range rate from doppler frequency shifts. Many of these systems are mobile sensors. Several, in combination, may be required to produce either a position vector for differentiation or a velocity vector for combination with position measured from other sources.

Data available from missile guidance systems, both inertial and radio, have gained in usage in range safety computations. Most radio guidance systems are either pulse

radars or CW systems as discussed above. Data from inertial guidance systems have found use primarily due to the lack of sufficient accuracy obtained through either pulse or CW systems. The use of telemetered inertial guidance data for range safety purposes is questionable from the standpoint that the system which is guiding a malfunctioning missile is also being used in the computations to determine whether it should be destroyed. Experience has shown that the earliest indication of a malfunction is often obtained through on-board telemetered measurements.

2.2.1.1 Instrumentation systems that are currently installed and operating at the ranges surveyed are identified in Table 1.

2.2.1.2 Few new instrumentation systems that can be used for range safety purposes are planned in the immediate future. For the most part, existing systems will continue in use and will be representative of future systems. Efforts in radar calibration are expected to result in improved tracking accuracies. In addition to inertial guidance data, increased range safety use of telemetered parameters such as guidance commands, nozzle deflection angles, and pitch, yaw, and roll angles and rates can be expected. Data from external tracking systems will be combined in the real-time computer with telemetered guidance data and missile body parameters to provide additional data for range-safety decision making. With upgraded computers, analytical models, and display systems, range safety visibility into real-time missile performance should improve correspondingly.

To provide accurate post-flight trajectory data for Trident missile launches on the Eastern and Pacific missile ranges, the missiles will be equipped with special translators that operate in conjunction with orbiting satellites and ground stations referred to as "pseudo satellites."

TABLE I

Installed Range Instrumentation Systems

DESCRIPTION	DATA OBSERVED	TRACKING MODE*	MANUFACTURER	RANGE**
A. Pulse Radars				
MOD II	R,A,E	T,S	REEVES	1
AN/FPS-16	R,A,E,$(\dot{R})^{2,3}$	T,S	RCA	1,2,3,4, 5,7
AN/FPQ-6	R,A,E,$\dot{R}$	T,S	RCA	2,5
AN/TPQ-18	R,A,E,$\dot{R}$	T,S	RCA	2,6
AN/MPS-25	R,A,E	T,S	RCA	3,4,7
MPS-19	R,A,E	T,S	REEVES	5
SPANDAR	R,A,E,$\dot{R}$	T,S		5
AN/FPQ-10	R,A,E	T,S	Sperry	4,7
AN/FPS-13	R,A,E	T,S	RCA	1
MPS-36	R,A,E,$\dot{R}$	T,S	RCA	2,3,6
AN/FPQ-14	R,A,E***	T,S	RCA	1
ALCOR	R,A,E****	T,S	RCA	6
B. CW Systems				
GERTS	R,A,E,$\dot{R}$,$\dot{P}$,$\dot{Q}$	T	GE	2
AME	X,Y,$\dot{Z}$	T		3
C. Guidance Systems				
Inertial	Position & Velocity Components	Telemetry		1,2,4
Radio	R,A,E,			2,6

*T = Transponder or beacon; S = Skin tracking.

**1 = DET 1/SAMTEC, 2 = SAMTEC, 3 = WSMR, 4 = PMTC, 5 = Wallops, 6 = KMR, 7 = ADTC.

$()^{X}$ Denotes additional capability existing at X range over and above the standard.

*** Data observed in R,A,E, but output in geocentric, earth fixed EFG$\dot{E}\dot{F}\dot{G}$ position and velocity.

****ARPA Lincoln C-band observables radar.

By adding other remote ranging stations, the system's application will be extended to include real-time range safety use. Four remote stations can be used simultaneously in a time-division-multiple-access mode with a master ranging station. The master and each remote station transmit signals in sequence to the missile where the signals are shifted in frequency by the missile translator and retransmitted to the master station. Here the signals are processed to extract a range sum and range-rate sum for each station. Missile position and velocity components are then computed from these sums.

2.2.2 Range safety computation and display equipment, both that in use and that planned for installation in the future, represents a variety of well known companies. Supervisory software techniques employed with computational equipment, while quite varied among the ranges surveyed, generally fall within two categories: (1) dedication of a computer system to the solution of range safety related problems is noted in the next two subparagraphs as dedicated (D); and (2) operation within a multiprogrammed and possibly multiprocessor environment which allows other unrelated data processing to continue concurrently with the real-time range safety calculation is noted in the next two subparagraphs as shared (S).

Display equipment in use for presenting range safety calculations are: plotters (P), digital displays (DD), cathode ray tubes (CRT), teletypes (TTY), and lighted matrices (LM). Lighted matrices generally are used to present instrumentation and measurement data status.

2.2.2.1 Presently Installed Systems

Range	Computer	Computer Usage	Display Equip.
SAMTEC	IBM-7044	D	P,LM
DET 1/SAMTEC	CDC-3600/CDC 3100	D	CRT,LM,DD
NASA/Wallops	HW-625	D,S	DD,P,CRT,TTY, LM
WSMR	Univac 1108	D,S	P,DD,LM,TTY
PMTC	Univac 1230/1212	D	P,LM
KMR	CDC 7600/PDP 11-05	S	CRT,LM
ADTC	CDC-6600/IBM 360-65/ PDP-15/PDP-11	D,S	CRT,P

2.2.2.2 Planned Future Systems

Range	Computer	Computer Usage	Display Equip.
SAMTEC	IBM 360-65 to be replaced by IBM 370-165	D	P,DD,CRT,LM
DET 1/SAMTEC	CDC-CYBER 74	D,S	CRT,LM,DD
NASA/Wallops	None		P,TTY,LM, expanded use of CRT
WSMR	Upgraded System Planned	D	P,DD,LM,CRT, TTY
PMTC	Upgraded System Planned	D	CRT (Color),P, DD,TTY,LM

3.0 THEORY OF FILTERING

3.1 The basic objective of filtering is to remove random noise from observations. This noise may disguise valid data

to a point where it cannot be used as an accurate measurement. The techniques used to accomplish this may vary in complexity from the averaging of data samples to the more complex derivation of state vectors for prediction of new measurements and recursive updating of the estimates. The most common techniques employed in the past have been the development of finite-length least-square filters. With the improved computational systems the filtering of data may be accomplished by more complex statistical techniques.

In the selection of a mathematical technique to provide the best determination of position, velocity, and acceleration one must always consider the limitations imposed by the computational equipment. This means that the technique must be adapted to the solution of the problem as well as the computing equipment to be used. Such adaptation is especially necessary when considering a filtering technique for use in real time to supply inputs to range safety calculations.

Restrictions imposed through implementing any computation in real time are: (1) computational time requirements and (2) computer storage requirements. Obviously, a sophisticated best estimate of trajectory scheme, which requires 64,000 words of computer storage and 1 second of computational time per cycle, cannot be implemented on a 32,000 word storage computer ten times per second.

3.2 Filtering Techniques

3.2.1 Data filtering or smoothing can be thought of as the replacement of a curve or a sequence of points, by another that is in some sense more regular, and yet whose ordinates, for any abscissa, are changed as little as possible. The irregularities in sequence of points may be due to errors in measurement. If theory requires the theoretically

correct points to lie on a given curve, one may apply some method of curve fitting, possibly least squares. If not, one may select arbitrarily a simple function, possibly a polynomial, and fit it by least squares. If the purpose is merely to obtain a smooth graph, this may be drawn visually. A somewhat more sophisticated method is to take, say, 5 consecutive points, fit a parabola, and replace the middle point by the one on the parabola. The next parabola requires four of these points and one new one.

3.2.2 Filtering techniques can be roughly divided into two basic groups as follows:

(1) Recursive filters may operate on a stored array of previous measurements or restrict calculations to the latest point. The principle of recursion as applied to data filtering relates the previous filter output to the current calculated values. Such a recursive filter cannot effectively operate without the previously calculated values. This implies that the filter must be supplied an approximation of these values for any initial filter start. Many schemes for supplying these initial approximations of position, velocity, and acceleration are in use and are known as filter initialization.

(2) Non-recursive filters generally operate on a stored array of raw data points. This type of filter requires only that the array be filled and updated as each new point is received. No history of previous filter output is required to generate the current values of position, velocity and acceleration.

The recursive and non-recursive properties of filters are sometimes difficult to categorize and therefore examples of each type are presented in the following subparagraphs.

3.2.2.1 The most simple non-recursive technique of filtering (smoothing) data is averaging points. This can be a simple average of data points to establish a mean or midpoint on a curve or a more complex weighted average designed to establish the latest value or end point. Average velocity can be determined by computing the average of position-first differences. Average acceleration can be determined by computing the average of position-second differences. The latest or real-time values of velocity and acceleration can be approximated by weighting the average. Many pre-filter data editing or filter initialization schemes use averaging techniques.

3.2.2.2 The approach of filtering (smoothing) the raw measurements often utilizes a "classical" least-squares technique. This technique simply utilizes the principle of fitting a given number of data points to a second- or third-degree polynomial so as to minimize the sums of the squares of the residuals between the polynomial and the measured data points. By making certain assumptions such as the number of data points, the output point, constant time intervals, etc., the least-squares technique can be resolved into a finite number of coefficients, called weights, which can then be used to scale the input data points to yield the desired output. For purposes of flight safety this point must be related to the most recent input value in order to avoid costly time delays. The above technique, however, does not consider such important information as vehicle capabilities, gravity models, and instrumentation noise estimates. In order to introduce these effects, the above techniques may be modified to make use of this information as described in the following subparagraphs:

3.2.2.3 A variety of formulations of the Kalman filter exist. They all provide a means for the optimum use of a set

of measurements to give the best (i.e., least squares) estimate of the state of a system and are based on the work of Dr. Rudolf E. Kalman. The Kalman filter may be expressed very generally as a set of matrix equations, but the content of the matrices will vary depending on the system to which the filter is applied. Many filters, however, are erroneously referred to as Kalman types which do not contain the essential optimal data weighting feature.

Those parameters of the system of interest which adequately describe its present dynamic conditions are defined as the state variables of the system. For example, the state variables for a missile in flight might be the three components each of position, velocity, and acceleration. These nine parameters would then be considered the coordinates of a nine-dimensional space, and the condition of the missile would be described by a nine-element state vector. Those parameters of the system which are measured are considered to be the elements of a measurement vector. For example, with a single conventional radar there would be a three-dimensional measurement vector with elements of range, azimuth and elevation.

The state of the system is assumed to change in some predictable manner so that future states may be predicted from the current state. Since measurements are made at fixed time intervals, the Kalman filter, like others in this report, works by discrete steps. It is possible then to write a state transition matrix such that the state vector at the next time point can be predicted by multiplying the current state vector by a state transition matrix. For the example system this is nothing more than a Taylor series extrapolation.

The observed quantities can be predicted from the predicted state, i.e., the observation vector is a function of the state vector. The actual observations are measured and the differences between actual and observed values are multiplied by an optimal weighting matrix to obtain the corrections which will update or improve the current estimate of the state vector.

Obtaining the optimal weighting matrix is the key to the Kalman filter. Briefly, the weights are derived by consideration of the covariance matrices of the observation and state vectors so that that state vector error is minimized in a least-squares sense.

4.0 INDIVIDUAL RANGE FILTERS

4.1 DET 1/SAMTEC (ETR)

4.1.1 Recursive PV Filter

4.1.1.1 Development and Usage

A recursive position and velocity (PV) filter is the first filter applied to the input raw data from the radars in azimuth, elevation and range. This filter, called SMW, is used to pre-edit the raw data by removing wild points from the input sequence. The smoothed position and velocity estimates are retained merely as inputs for the next call. Data received by the computer from the radars can be made completely erroneous by errors in transmission. If such data were allowed into the velocity filter, the computer velocity would be bad for as long as the point stayed in the filter (normally 5 seconds). This in turn would cause an erroneous impact point to be computed. Since the calculations require that the data rate be maintained,

the edited point must be replaced by an estimate. For this purpose a light linear filter seemed most desirable.

4.1.1.2 Theory and Method of Operation

A more complete discussion of the theory of this form of filtering can be found in Section 5, Reference 13; a synopsis of this discussion follows. We define the following parameters:

$\{X(nT)\} = \{X_n\}$ = Raw input sampled data sequence. Sampling is at equal time intervals T.

$\{W(nT)\} = \{W_n\}$ = "Signal" or desired component of the input sequence $\{X_n\}$.

$\{y(nT)\} = \{y_n\}$ = "Noise" or undesired component of the input sequence $\{X_n\}$.

And we make the following assumptions:

a. Let $X_n = W_n + y_n$ by definition

b. The expected value of the noise sequence y_n is zero, and its autocorrelation function is assumed to have the form

$$\overline{y_i y_j} = \sigma^2 \text{ if } i = j$$
$$= 0 \text{ if } i \neq j$$

c. The cross correlation between the signal W_n and the noise y_n is assumed to be zero.

It is desired to generate a new sequence P_n as a function of the input sequence X_n by means of a linear time invariant transformation in such a way that the new sequence is a "best" approximation to the signal W_n or some linear function of W_n. Hence, P_n must be of the form

$$P_n = \sum_{k=o}^{\infty} g_k X_{n-k} \tag{A-1}$$

which is a convolution in which g_k is the desired filter unit impulse response. Alternatively (A-1) may be written as

$$P(Z) = G(Z)X(Z), \tag{A-2}$$

where P(Z) is the Z transform of P_n, X(Z) is the Z transform of the input sequence X_n and G(Z) is the Z transform of g_k or the desired filter transfer function.

As a form for a linear transformation, let

$$g_{nu} = \sum_{k=o}^{\infty} a_k W_{n-k} \tag{A-3}$$

be an arbitrarily chosen linear function of the signal W_n which it is desired to estimate by means of P_n, and let k_o be the ratio of steady-state variance in P_n to the variance in raw input.

The variance in P_n is given by:

$$\sigma_P^2 = \overline{(P_n)^2} - (\overline{P}_n)^2 = \sigma^2 \sum_{k=o}^{\infty} g_k^2 \tag{A-4}$$

Therefore, from assumption b,

$$K_o = \sum_{k=o}^{\infty} g_k^2 = \sigma_p^2/\sigma^2 \tag{A-5}$$

Let

$$D^2 = \sum_{n=o}^{\infty} (n - \sum_{k=o}^{\infty} g_k(n-k))^2 \tag{A-6}$$

The "best" filter is defined to be the one which for a given K_o will make D^2 a minimum or for a given D^2 will make K_o a minimum. K_o is a measure of the ability of the filter to reduce the random noise in its output. D^2 is a measure of the ability of the filter to respond to a signal input, i.e., it is essentially a measure of response.

It is therefore necessary to minimize:

$$J = K_o + \lambda D^2 \tag{A-7}$$

where λ is the Lagrange multiplier. Substituting the values for K_o and D^2 in (A-7) yields

$$J = \sum_{k=o}^{\infty} g_k^2 + \lambda \left(\sum_{n=o}^{\infty} n - \sum_{n=o}^{\infty} g_k(n-k) \right)^2 \tag{A-8}$$

The optimum position filter transfer function, G(Z), is found to be

$$\overline{G}(Z) = \frac{Z(\alpha Z + \beta - \alpha)}{Z^2 - (2-\alpha-\beta)Z + (1-\alpha)} \tag{A-9}$$

The velocity filter transfer function, $\dot{G}(Z)$, is given by

$$\dot{G}(Z) = \left(\frac{1}{T}\right) \frac{\beta Z(Z-1)}{Z^2 - (2-\alpha-\beta)Z + 1 - \alpha} \tag{A-10}$$

where

$$\alpha = 1 \quad Z_1 Z_2 \tag{A-11.1}$$

$$\beta = (1-Z_1)\ (1-Z_2); \tag{A-11.2}$$

and Z_1, Z_2 are the roots of the quartic,

$(Z-1)^4 + \lambda T^2 Z^2 = 0$, which lie inside the unit circle in the Z plane. (A-11.3)

The recursive filter set of equations is given by

$$\delta_n = X_n - X_{Pn}, \tag{A-12.1}$$

$$\bar{X}_n = X_{Pn} + \alpha\delta_n, \tag{A-12.2}$$

$$\dot{X}_n = \dot{X}_{n-1} + \beta/T\delta_n, \tag{A-12.3}$$

$$X_{Pn+1} = \bar{X}_n + T\dot{X}_n \tag{A-12.4}$$

To edit, δ_n is compared in absolute value with a fixed edit limit, which is approximately 0.5 degree in angle and 400 feet in range. If it exceeds the limit, δ_n is replaced by zero, and X_n is replaced by X_{Pn}.

4.1.1.3 Outstanding Features

Initialization

This filter contains a routine for self-initialization. Based on an input counter, the first two points are differenced for the initial estimate of velocity and the second point is used for the initial smoothed position estimate.

Consecutive Edits

If five consecutive points are edited, the filter resets the initialization counter to call for self-initialization on the next pass through the filter.

4.1.1.4 Limitations

Since this filter does not compute acceleration, rapid changes in velocity will cause it to edit and reinitialize.

4.1.2 Recursive PVA Filter

4.1.2.1 Development and Usage

A recursive position, velocity, and acceleration (PVA) filter is next applied to the edited output from SMW. This filter, called BSMW, is used to develop estimates of the amount of noise in the data. These noise estimates, projected to impact, are later used as a criterion for selecting the radar used for the impact calculations. For the purpose of obtaining these noise estimates, a function was needed which could reasonably describe the motion of the vehicle over short periods of time. The quadratic was chosen and the recursive form was selected for savings in time and core. Light filtering is used in order to minimize the problems associated with either a staging or a malfunctioning missile.

4.1.2.2 Theory and Method of Operation

The following is an additional synopsis of the discussion found in Section 5, Reference 13. Using the same definitions and assumptions given for the PV filter, we then let

$$D^2 = \sum_{n=o}^{\infty}\left(\sum_{k=o}^{\infty} g_k W_{n-k} - g_n\right)^2 = \sum_{n=o}^{\infty}\left(\sum_{k=o}^{\infty} g_k W_{n-k} - \sum_{k=o}^{\infty} a_k W_{n-k}\right)^2 \qquad \text{(A-13)}$$

The function to be minimized, J, becomes in this case

$$J = \sum_{k=o}^{\infty} g_k^2 + \lambda \sum_{n=o}^{\infty}\left(\sum_{k=o}^{\infty} g_k W_{n-k} - \sum_{k=o}^{\infty} a_k W_{n-k} \right)^2 \tag{A-14}$$

The optimum PVA filter transfer functions are then

$$\bar{G}(Z) = Z[\alpha Z^2 - (2\alpha-\beta-\gamma)Z + \alpha + \gamma-\beta] / [Z^3 - (3-\alpha-\beta-\gamma)Z^2 - (2\alpha+\beta-\gamma-3)Z - (1-\alpha)]; \tag{A-15.1}$$

$$\dot{G}(Z) = Z(Z-1)\,(\beta Z + 2\gamma-\beta)/T[Z^3 - (3-\alpha-\beta-\gamma)Z^2 - (2\alpha+\beta-\gamma-3)Z - (1-\alpha)] \tag{A-15.2}$$

$$\ddot{G}(Z) = 2\gamma Z(Z-1)^2/T^2\,[Z^3 - (3-\alpha-\beta-\gamma)Z^2 - (2\alpha+\beta-\gamma-3)Z - (1-\alpha)]; \tag{A-15.3}$$

For any value chosen for γ, the roots of $(Z-1)^6 - \lambda T^4 Z^3 = 0$ which lie inside the unit circle in the Z plane, namely $Z_1 Z_2 Z_3$, are computed. The constants α, β, γ may be computed by the relations:

$$\alpha = 1 - Z_1 Z_1 Z_3; \tag{A-16.1}$$

$$\beta = \tfrac{1}{2}\,[Z_1 Z_2 Z_3 + -Z_1 - Z_2 - Z_3 - Z_1 Z_2 - Z_3(Z_1 + Z_2)]; \tag{A-16.2}$$

and

$$\gamma = \tfrac{1}{2}(1-Z_1)\,(1-Z_2)\,(1-Z_3). \tag{A-16.3}$$

Inversion of the set (A-15) yields the following set of filter equations in the time domain:

$$\delta_n = X_n - XP_n; \tag{A-17.1}$$

$$\bar{X}_n = XP_n + \alpha\delta_n \tag{A-17.2}$$

$$\dot{X}_n = \dot{X}_{n-1} + T\ddot{X}_{n-1} + \beta/T\ \delta_n; \tag{A-17.3}$$

$$\ddot{X}_n = \ddot{X}_{n-1} + 2\gamma\delta_n/T^2; \tag{A-17.4}$$

$$XPn{+}1 = \bar{X}_n + T\dot{X}_n + T^2\ddot{X}_n/2 \tag{A-17.5}$$

This filter contains an edit capability of the same form as that of SMW, but since a very large edit limit is specified, this capability is not used.

4.1.2.3 Outstanding Features

This filter also contains a routine for self-initialization. Based on an input counter, the first three points are second-differenced for an initial acceleration estimate, the second and third points are differenced for velocity, and the third point is used as the initial smoothed position estimate.

4.1.2.4 Limitations

Rapid changes in acceleration can cause this filter to either load or lag the raw data, leading to an apparent increase in the noise in the data.

4.1.3 Finite Span Velocity Filters

4.1.3.1 Development and Usage

In order to develop a velocity estimate for impact prediction, the data from the selected radar is passed through one of three filters: GRSM46 (51 point, half-second delayed, for 10 pps data); GRSMPP (51 point, midpoint, for 10 pps data); or G2SM00 (21 point, midpoint, for 20 pps data). In order to perform this task in a minimum amount of time, it was desirable to take advantage of a special capability of the machine which allows the binary exponent of a floating point number to be altered. This operation effectively multiplies or divides numbers by powers of two at a considerable time saving over floating point multiplication.

Therefore, a method which consisted of a linear combination of data points utilizing powers of two was sought as a substitute for a full least-squares solution. The finite span method was chosen over the recursive in order to get a heavier filter without the overshoots and settling problems inherent in the recursive. This also gives a fixed delay in the output, whereas the recursive is generally considered to have a variable delay.

4.1.3.2 Theory and Method of Operation

We define the following parameters:

X_n = Input data sequence sampled at equal time intervals T;

W_n = "Signal," or desired *component* of X_n;

Y_n = "Noise," or undesired component of X_n.

And we make the following assumptions:

$$X_n = W_n + Y_n;$$

$$W_{n+k} = W_n + kT\mathring{W}_n + \frac{k^2T^2}{2}\ddot{W}_n, \quad (-N\leq k\leq P);$$

The least-squares fit for velocity can be expressed in this form:

$$\dot{W}_n \approx \dot{X}_n = \sum_{k=-N}^{k=P} g_k X_{n+k}.$$

These three filters are all designed to be used with double-stored tables so that the tables do not need shifting as successive points are added. The starting location of the double-stored table and the index to the oldest point in the table is passed to the routine.

4.1.3.3 Outstanding Features

These three filters are notable for their speed, both in required overhead and actual computation.

4.1.3.4 Limitations

The main drawback of these filters is the storage required for the tables - 70 cells for each coordinate of the double-stored table.

4.1.4 Variable Edit Filter

4.1.4.1 Development and Usage

This filter was developed in conjunction with the orbit phase real-time program at ETR. For orbital passes which required high elevation angles of the radars and consequently rapid changes of velocity and acceleration in azimuth, the combination of SMW and BSMW proved to have inherent drawbacks. It seemed desirable then to transform the data first to a Cartesian coordinate system before attempting to edit or filter it. Once the data were transformed, the next problem was to establish a meaningful edit limit. For short ranges, a given angular error produced a smaller error in the Cartesian system than it would for a longer range. There are also effects from target geometry which cause fluctuations in the noise level. Consequently, a fixed percentage of the data was edited by allowing the edit level to vary. It became apparent that this edit level was then a measure of the noise in the data and could be used as a criterion for radar selection. Since this scheme replaced the SMW-BSMW combination and several routines for error averaging and transforming to the Cartesian system, a considerable time savings was realized as a fortuitous by-product.

4.1.4.2 Theory and Method of Operation

We define the following parameters:

R = Ratio of editing desired;

E = Number of points edited;

P = Number of data points processed;

L = Edit limit.

We then can compute the following quantities:

$$C = E/P = \text{Ratio of editing achieved} \quad \text{(B-1)}$$

$$M = C/R = \text{Edit-limit multiplier} \quad \text{(B-2)}$$

The smoothing scheme employed is the same as that of BSMW, the recursive PVA filter. The position difference is computed as in equation (A-17.1), and then compared in the absolute value to the edit limit. This is done for all three position coordinates.

If the edit limit is exceeded, the data point is edited in all three coordinates. Once the point is edited, the edit limit multiplier is checked to ensure it is greater than 1.0 and then is used to adjust the edit limit.

$$L = L \cdot M \quad \text{(B-3)}$$

If the point is not edited, the edit limit multiplier is checked to ensure it is less than 1.0 and then is used to adjust the edit limit as in equation (B-3).

4.1.4.3 Outstanding Features

In order to keep the edit limit multiplier M from becoming unaffected by an edit, a maximum is set upon the value to which P, the number of data points processed, is allowed to go. When it reaches this limit, it is reset to its minimum and E, the number of points edited, is adjusted so that their ratio C remains essentially the same. This process maintains a desired level of "responsiveness" in the edit level itself. If several points in a row are edited, the edit level will then increase in an almost exponential manner until editing ceases.

4.1.4.4 Limitations

This filter contains no internal logic for reinitialization. If it is desired to reset the filter, the point counter must be set to zero externally.

4.2 Space and Missile Test Center

4.2.1 Exponential filters are currently used in the primary flight safety computer (IBM 7044) and the backup flight safety computer (CDC-3300). The same basic filter will also be used in the IBM 360-65 computer when it replaces the IBM 7044 as the primary flight safety computer.

4.2.1.1 Exponential filters were incorporated into the Point Arguello Range Safety Impact Predictor, the predecessor to the Vandenberg Impact Prediction System (VIPS), in June 1965. These filters were then carried on into VIPS when the program was renamed after reprogramming for the IBM 7044 in late 1966. The filtering is accomplished in two subroutines in VIPS. They are EDIT and EDIT2. EDIT is used to (1) compute the mean-average deviation between the predicted

value and the measured value, (2) edit wild data points from raw data, and (3) smooth the polar coordinates. EDIT2 is used to (1) smooth geocentric position and (2) differentiate this position to obtain geocentric velocity.

VIPS is the primary real-time computer program which supports all major launches from Vandenberg AFB, California. VIPS operates on the IBM 7044. The Backup Impact Prediction System (BIPS) is the secondary real-time computer program which operates on the CDC-3300 computer. BIPS is equivalent to VIPS except for automatic abort, number of input sensors and other aspects. The filters in both systems are basically identical. Radar measurements, measurements from the General Electric Radio Tracking System (GERTS) and data extracted from telemetered inertial guidance systems are used as inputs into VIPS. These data are checked for site identification, parity and on-target indicators to determine data quality. These data are then used to compute impact prediction, present position and display information for the Missile Flight Control Officer (MFCO).

The data required to compute impact prediction for a missile are (1) the position vector and (2) the velocity vector of the missile. Three independent components of position and three independent components of velocity meet these requirements. These data are obtained by direct measurement from the GERTS and from telemetered inertial guidance system measurements. GERTS position data is edited to remove wild data spikes and smoothed to reduce the noise on the measurements. GERTS velocity measurements are used in raw form without either smoothing or editing. All six parameters from telemetered inertial guidance are used directly and are not smoothed, although gross editing of the inertial guidance prior to input into the impact prediction routine is performed.

Most radar measurements consist of the three components of position only and other means of producing velocity must be used. Currently, these position measurements are edited, smoothed and a cross-range error computed for use in source selection. The smooth polar position components are then transformed into a geocentric coordinate system. The geocentric positions are then smoothed and independently differentiated to produce velocity for use in impact prediction and acceleration for use in prediction for updating position and velocity. The impact prediction is then corrected for the effects of drag and earth oblateness. The final predicted impact point is then displayed for the MFCO.

GERTS data (R, A, E, $\dot{R}$, $\dot{P}$, $\dot{Q}$) are input and the position data are edited and smoothed in subroutine EDIT. The range (R_o) from the central rate station to the object is computed and edited.

The ranges (R_1 and R_2) to the other two rate stations are computed. These are differenced to obtain P and Q.

$$P = R_o - R_1$$

$$Q = R_o - R_2$$

These quantities are then edited and differentiated to compute $\dot{R}_o$, $\dot{P}$, $\dot{Q}$, respectively in subroutine EDIT. If the raw input range rate R is not good, the R_o computed in EDIT is substituted. If the raw lateral rates P and $\dot{Q}$ are good they are used in further computations; however, if the raw $\dot{P}$ and $\dot{Q}$ are not good, then the values computed in EDIT are substituted. Geocentric quasi-inertial position and velocities are then computed to be used in impact prediction computations.

4.2.1.2 The inputs for EDIT are:

a. Raw data argument R azimuth or elevation for radar.

b. Data quality Flag - either good or bad.

c. Initial estimates of sigma - (IMAD) from a priori estimates.

There is one filter for each of the raw data arguments, i.e., R, A and E for each radar being input.

The number of samples of filter initialization (COUNT) is tested against an optimum number (LIM1). If COUNT is less, the edit filter undergoes initialization. During initialization, COUNT is incremented and smoothing constants α and β are updated.

$$COUNT = COUNT + 1$$

$$\alpha = \frac{1}{COUNT} \text{ as COUNT approaches LIM1}$$

$$\beta = 1 - \alpha$$

Also, if COUNT is less than three, no mean-average deviation updating or data predicting is allowed. If COUNT equals LIM1, filter initializing is bypassed and editing is allowed. If COUNT is greater, the edit filter undergoes reinitialization. During reinitialization, COUNT is reset to LIM1 (value - 20), and new smoothing operators are calculated:

$$S_n^1 = X_{n-1} - V_{n-1} \left(\frac{\beta}{\alpha}\right)$$

$$S_n^2 = X_{n-1} - V_{n-1} \left(\frac{2\beta}{\alpha}\right)$$

$$S_n^3 = X_{n-1} - V_{n-1} \left(\frac{3\beta}{\alpha}\right)$$

where

$$COUNT = LIM1$$

$$\alpha = \frac{1}{COUNT}$$

$$\beta = 1 - \alpha$$

If COUNT is greater than three, the predicted value for the current sample is calculated:

$$X_{pred} = X_{n-1} + V_{n-1} + An-1$$

This predicted value is compared with the raw input value and a forecast deviation (DELXYZ) is calculated. If this forecasted deviation equals zero, editing and mean-average deviation updating is bypassed and data smoothing is performed. Three sigma is calculated and compared with the current forecast deviation. If the forecasted deviation is greater, the raw data is replaced by the predicted value. If the filter is being initialized (COUNT is less than LIM1), editing is bypassed and only the mean-average deviation is updated.

$$\sigma = \beta\sigma_{n-1} + \Delta X\alpha \text{ Mean-Average Deviation}$$

where

$$\Delta X = |X_{pred} - X_{raw}| \text{ Forecasted deviation between}$$

predicted and raw input data.

If editing is performed, the forecasted deviation is compared with 20 sigma. If the deviation is greater than

20 sigma, the mean-average deviation remains unchanged and the edit counter (TALLY) is incremented. After the raw data is thoroughly evaluated, data smoothing is initiated. New smoothing operations for the current data sample are calculated:

$$S_n^1 = \alpha X_n + \beta S_{n-1}^1$$

$$S_n^2 = \alpha S_n^1 + \beta S_{n-1}^2$$

$$S_n^3 = \alpha S_n^2 + \beta S_{n-1}^3$$

These operations are linearly combined to compute smoothed position, velocity and acceleration.

$$X_n = 3(S_n^1 - S_n^2) + S_n^3 \quad \text{smoothed argument}$$

$$V_n = \frac{\alpha}{2\beta 2} [(6-5a)\ S_n^1 - (10 - 8\alpha)\ S_n^2 + (4-3\alpha)\ S_n^3] \quad \text{smoothed velocity of argument}$$

$$\frac{A_n}{2} = \frac{\alpha^2}{4\beta 2} (S_n^1 - 2S_n^2 + S_n^3) \quad \text{smoothed acceleration of argument}$$

The smoothed acceleration will then be used in calculating predicted position (X_{pred}) for the next sample.

EDIT2 - The inputs to this subroutine consist of:

a. Quasi-inertial position (XI, ETA, ZETA)

b. Filter weight factor (α).

The outputs consist of:

a. Smoothed quasi-inertial position (XIs, ETAs, ZETAs)

b. Quasi-inertial velocity (XIDOT, ETADOT, ZETADOT)

c. Quasi-inertial acceleration components.

α = Filter weight

$\beta = 1 - \alpha$

$S_n^* =$ Argument

$$S_{n-1}^1 = X_{n-1} - \frac{\beta V_{n-1}}{\alpha}$$

$$S_{n-1}^2 = X_{n-1} - \frac{2\beta V_{n-1}}{\alpha}$$

$$S_{n-1}^3 = X_{n-1} - \frac{3\beta V_{n-1}}{\alpha}$$

Data is smoothed by the following:

$S_n^1 = S_{n-1}^1 \beta + S_n^* \alpha$ First operator

$S_n^2 = S_{n-1}^2 \beta + S_n^1 \alpha$ Second operator

$S_n^3 = S_{n-1}^3 \beta + S_n^2 \alpha$ Third operator

$X_n = 3(S_n^1 - S_n^2) + S_n^3$ Smoothed argument

$V_n = \{(8\alpha-10) S_n^2 - (3\alpha-4) S_n^3 - (5\alpha-6) S_n^1\} \dfrac{\alpha}{2\beta\ 2}$ Smoothed velocity of argument

$A_n = (S_n^1 - 2 S_n^2 + S_n^3) \alpha^2 / \beta^2$ Smoothed accelerator of argument

4.2.1.3 The exponential is flexible in that the amount of smoothing can be changed readily to minimize noise transmitted through the filter. It is important, however, that an appropriate compromise be made between how much noise is passed through the filter and how fast the filter is able to respond to the data. The method allows for (1) efficient data editing, (2) prediction for updating, (3) data smoothing, (4) data differentiation, and (5) an evaluation of individual sensor data quality for use in source selection.

4.2.1.4 The limitations of this type of filter are (1) it is fairly difficult to initialize and (2) it is not responsive to rapid changes in acceleration.

4.3 KWAJALEIN MISSILE RANGE

4.3.1 Kwajalein Range Safety System Filter

The Kwajalein Range Safety System (KRSS) is designed to provide a range safety capability which can be applied to those airborne vehicle test programs which require range safety support by the Kwajalein Missile Range (KMR). The filter developed for the KRSS is presented herein. The characteristics of this filter are described, and the method adopted for selecting the optimum operating point is given in detail.

The KRSS filter consists of an expanding memory polynomial filter of degree two for the purposes of "state vector initialization" followed by a fading memory polynomial filter of degree two after "state vector initialization." There are a number of reasons for choosing this filter design.

a. This filter combination virtually eliminates initialization errors. This is because the algorithm is completely self-starting. After precisely 3 cycles the initial values will have been completely dropped.

b. There is an excellent theoretical basis for the filter equations which are rigorously derived in Section 5.0, Reference (13). Closed form expressions for the variance reduction factor (VRF) associated with position, velocity, and acceleration are available for both the expanding and fading memory algorithms.

c. The expanding memory polynomial filter combined with the fading memory polynomial filter can be formulated as a set of extremely compact recursive algorithms.

d. The Laguerre discrete orthogonal polynomials form the basis for the fading memory filter, and have an exponential weight function θ^X which "fades" out in a well behaved manner as X increases. This parameter θ is useful in controlling the balance between random noise suppression and filter dynamic lag and in characterizing filter behavior.

e. The filter design facilitates the optimization of the filter weights during real-time operation for a variety of range safety applications.

4.3.2 KRSS Filter Description

The KRSS filter equations consist of an expanding memory polynomial filter of degree two for "state vector initialization" and an exponentially fading memory polynomial filter of degree two after "state vector initialization" has been completed. The equations are rigorously derived in Section 5.0, Reference (13) and are structured for recursive use.

4.3.2.1 Filter Equations

The filter equations are presented in the following paragraphs:

The residual is given by:

$$\varepsilon_x(i) = x(i) - \hat{x}(i/i-1); \quad x \rightarrow y,z \qquad (1)$$

The current state vector estimate is given by the following:

$$\left.\begin{aligned} \hat{\ddot{x}}(i/i) &= \hat{\ddot{x}}(i/i-1)+(2\gamma/\tau^2)\varepsilon_x(i) \\ \hat{\dot{x}}(i/i) &= \hat{\dot{x}}(i/i-1)+(\beta/\tau)\varepsilon_x(i) \\ \hat{x}(i/i) &= \hat{x}(i/i-1)+(\alpha)\varepsilon_x(i) \end{aligned}\right\} x \rightarrow y,z \qquad (2)$$

The 1-step prediction estimate of the state vector is given by the following:

$$\left.\begin{aligned}
\hat{x}(i+1/i) &= \hat{x}(i/i) \\
\hat{\dot{x}}(i+1/i) &= \hat{\dot{x}}(i/i)+(\tau)\hat{\ddot{x}}(i/i) \\
\hat{x}(i+1/i) &= \hat{x}(i/i)+(\tau)\hat{\dot{x}}(i/i)+\left(\frac{\tau^2}{2}\right)\hat{\ddot{x}}(i/i)
\end{aligned}\right\} \; x \rightarrow y,z \qquad (3)$$

These recursive equations give the filter estimate of the state vector ($\overline{X}$, $\overline{V}$, $\overline{A}$) evaluated for the current point (Equation 2), and for a 1-step prediction (Equation 3), are valid for both the expanding memory and the fading memory algorithms. This is convenient, since the only change which occurs in the transition from expanding memory to fading memory filter is the manner in which the weighting coefficients (α, β and γ) indicated in Equation (2) are calculated.

For expanding memory operation, α, β and γ are functionally related to the number of cycles of data processed, N=0,1,2... For fading memory operation, α, β and γ are functionally related to the filter control parameter θ. These functional relationships will be defined below.

The following definitions apply to Equations (1), (2) and (3):

x(i) = Measured x position at time t_i (raw inputs received at current time).

The symbol (^) over the variable denotes that the variable is an estimate derived from the filtering process. The double subscript index shown in Equations (1), (2) and (3) are defined as follows: the first index refers to the instant of validity of the variable estimate. The second

index refers to most recent data utilized in obtaining the estimate. For example,

$\hat{x}(i/i-1)$ = Smooth estimate of x position for time t_i obtained from data taken up to and including that at time t_{i-1}.

$\hat{x}(i/i)$ = Smooth estimate of x position for time t_i obtained from data taken up to and including that at time t_i.

$\hat{x}(i+1/i)$ = Predicted estimate of x position for time t_{i+1} obtained from data taken up to and including the current time t_i.

The velocity and acceleration estimates are defined in a similar fashion, utilizing double indexes to denote estimate validity time and data validity time.

τ = Time interval between data cycles.

$\varepsilon_x(i)$ = Residual computed during current cycle.

The weight coefficients α, β and γ of the expanding memory filter are functionally related to N, the number of cycles of data processed, by the following equations:

$$\alpha(N) = \frac{3(3N^2+3N+2)}{(N+3)\ (N+2)\ (N+1)} \quad \text{(position)} \tag{4}$$

$$\beta(N) = \frac{18(2N+1)}{(+3)\ (N+2)\ (N+1)} \quad \text{(velocity)} \tag{5}$$

$$\gamma(N) = \frac{30}{(N+3)\ (N+2)\ (N+1)} \quad \text{(acceleration)} \tag{6}$$

With N taking on successively the values 0,1,2,3,... N_m-1, where N_m is the value of the counter at which switching takes place from the expanding memory filter to the fading memory filter.

The weight coefficients α, β and γ of the fading memory filter are functionally related to the fading memory filter control parameter θ by the following equations:

$$\left.\begin{aligned} \alpha(\theta) &= 1-\theta^3 \qquad &(7)\\ \beta(\theta) &= 3/2(1-\theta)^2(1+\theta) \qquad &(8)\\ \gamma(\theta) &= \tfrac{1}{2}(1-\theta)^3 \qquad &(9)\end{aligned}\right\} \; 0 < \theta < 1$$

4.3.2.2 Variance Reduction Factor

4.3.2.2.1 Expanding Memory

The VRF for acceleration for an expanding memory filter is given by:

$$\text{VRFA} = \frac{720}{\tau^4(N+3)(N+2)(N+1)N(N-1)} \qquad (10)$$

This factor is independent of where the evaluation is made (current, 1-step, etc.). This is because of the choice of a second degree polynomial which implicitly assumes that the acceleration being estimated is constant. Once estimated for a set of data points, its estimate and the associated VRF is independent of when the evaluation is made.

The VRF for velocity is dependent upon where the evaluation is made. For a 1-step prediction this is given by:

$$\mathrm{VRFV}(1) = \frac{192N^2+744N+684}{\tau^2(N+3)(N+2)(N+1)N(N-1)} \tag{11}$$

The "current point" VRF for velocity has a slightly different form, given by:

$$\mathrm{VRFV}(0) = \frac{192N^2+24N-36}{\tau^2(N+3)(N+2)(N+1)N(N-1)} \tag{12}$$

Figure 1 shows the VRF for velocity for both the current point and for a 1-step prediction, as indicated by Equations (11) and (12).

Finally, the VRF for position is even more dependent upon where the evaluation is made than for velocity. For a 1-step prediction, the position VRF is given by:

$$\mathrm{VRFP}(1) = \frac{9N^2+27N+24}{(N+1)N(N-1)} \tag{13}$$

and the current state VRF for position is given by:

$$\mathrm{VRFP}(0) = \frac{9N^2+9N+6}{(N+1)N(N-1)} \tag{14}$$

4.3.2.2.2 Fading Memory

In fading memory weight Equations (7), (8) and (9) the parameter θ determines the weight given to past data, and therefore controls the tradeoff between dynamic response and noise suppression capability. A large value of θ produces good noise suppression, but poor dynamic response, whereas, a small θ gives a good dynamic response, but has low noise suppression. Figures 2, 3 and 4 show graphically the dependence of α, β and γ on the filter control parameter θ.

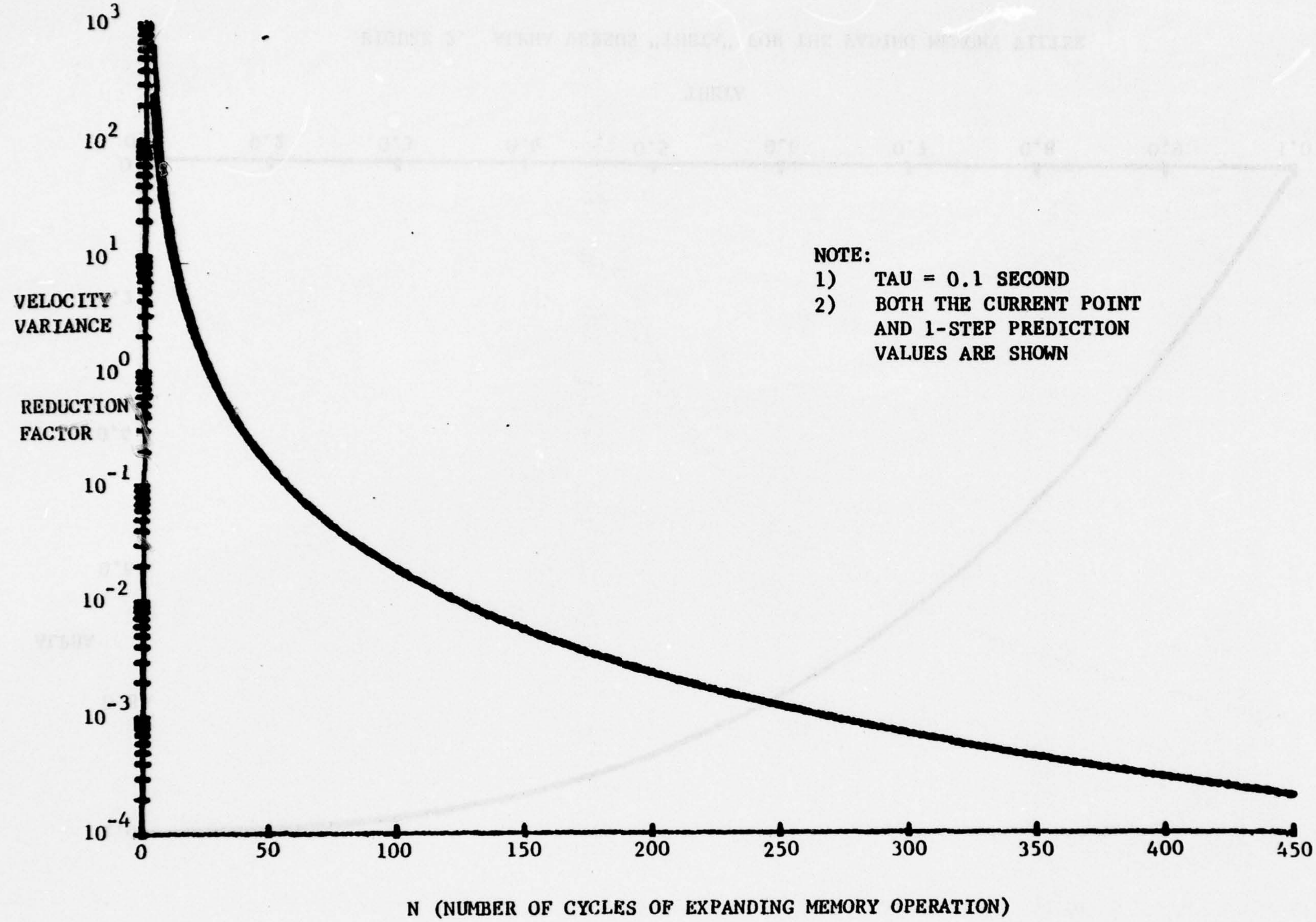


FIGURE 1. VELOCITY VARIANCE REDUCTION FACTOR VERSUS "N" FOR EXPANDING MEMORY FILTER

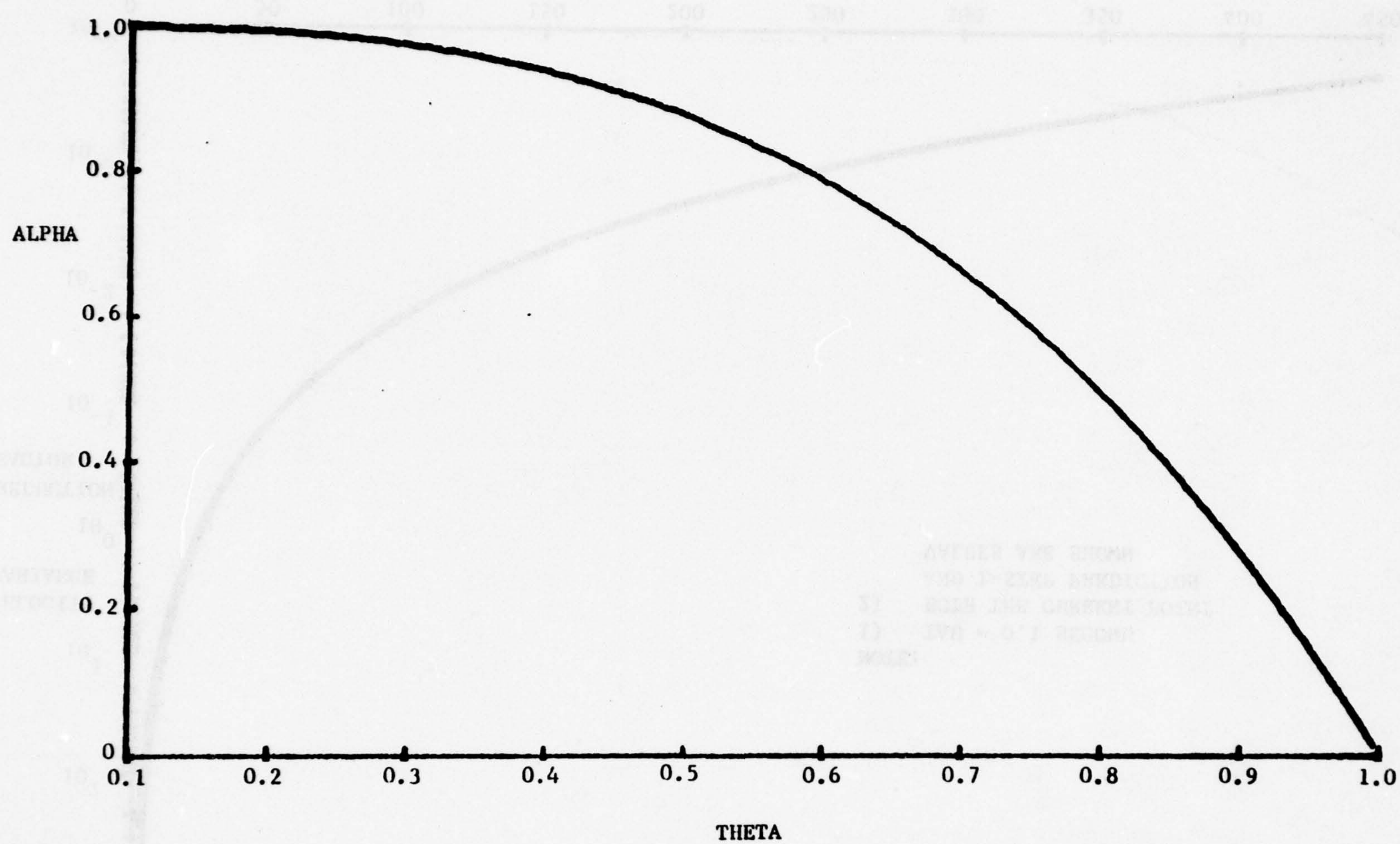


FIGURE 2. ALPHA VERSUS "THETA" FOR THE FADING MEMORY FILTER

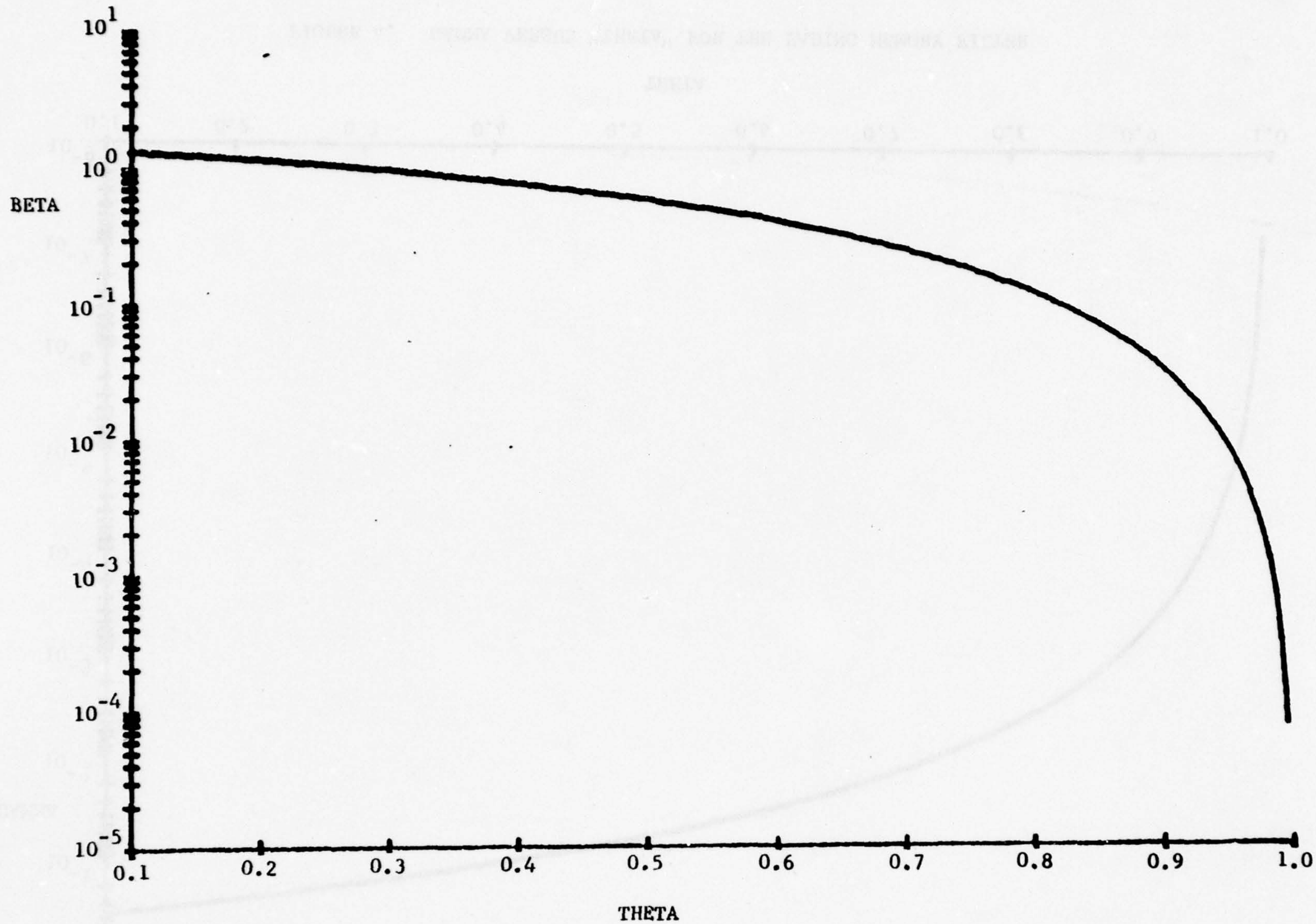


FIGURE 3. BETA VERSUS "THETA" FOR FADING MEMORY FILTER

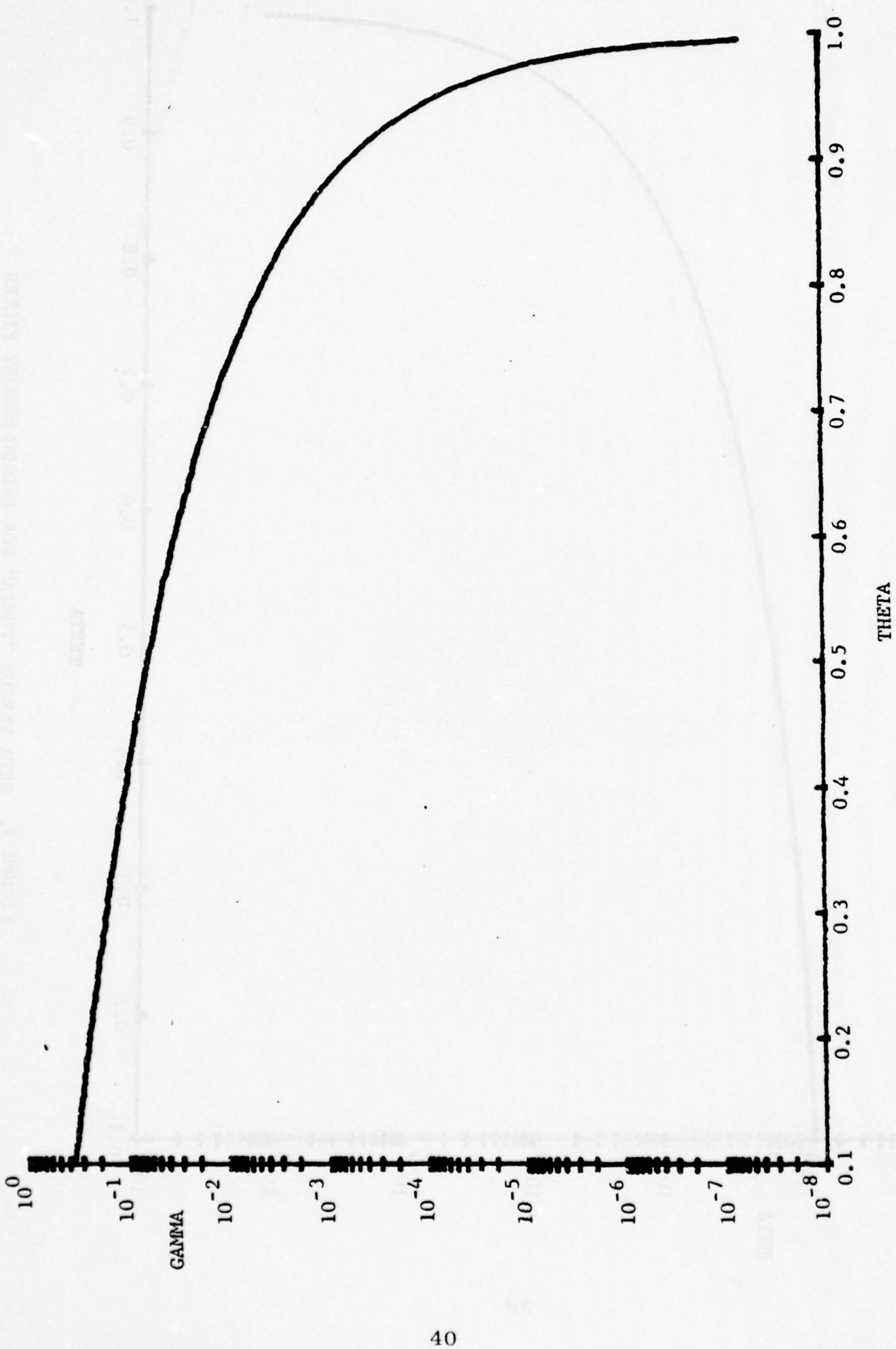


FIGURE 4. GAMMA VERSUS "THETA" FOR THE FADING MEMORY FILTER

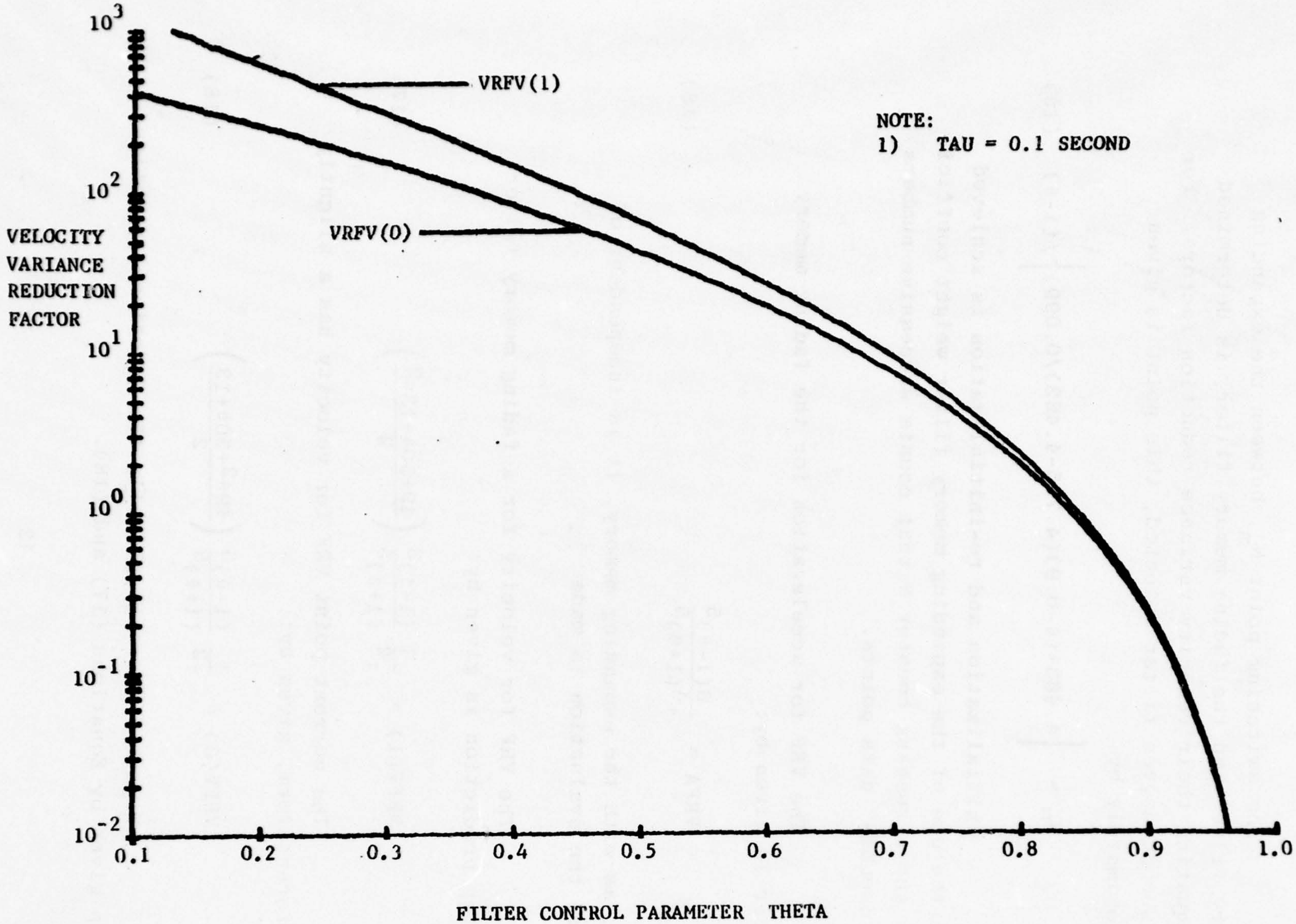


FIGURE 5. VELOCITY VARIANCE REDUCTION FACTOR VERSUS "THETA" FOR FADING MEMORY FILTER

The switching point N_m, between the expanding memory filter and the fading memory filter, is determined by equating their velocity variance reduction factors. For the second degree filter selected, this point is given approximately by:

$$N_m = \left\{4.463+(\theta-0.9)(4.785-4.463)/0.099)\right\}/(1-\theta) \quad (15)$$

Initialization and re-initialization is achieved with the use of the expanding memory filter weight coefficients and a running counter N that counts successive numbers of "useable" data points.

The VRF for acceleration for the fading memory filter is given by:

$$VRFA = \frac{6(1-\theta)^5}{\tau^4(1+\theta)^5} \quad (16)$$

and, as with the expanding memory, it is independent of where the evaluation is made.

The VRF for velocity for a fading memory filter 1-step prediction is given by:

$$VRFV(1) = \frac{1}{\tau^2}\frac{(1-\theta)^3}{(1+\theta)^5}\left(\frac{49+50\theta+13\theta^2}{2}\right) \quad (17)$$

The current point VRF for velocity has a slightly different form, given by:

$$VRFV(0) = \frac{1}{\tau^2}\frac{(1-\theta)^3}{(1+\theta)^5}\left(\frac{49\theta^2+50\theta+13}{2}\right) \quad (18)$$

Figure 5 shows graphically the functional relationship given by Equations (17) and (18).

Finally, the VRF for position for a 1-step prediction is given by:

$$VRFP(1) = \frac{1-\theta}{(1+\theta)^5} \quad (19+24\theta+16\theta^2+6\theta^3+\theta^4) \tag{19}$$

and the VRF for position evaluated at the current point is given by:

$$VRFP(0) = \frac{1-\theta}{(1-\theta)^5} \quad (19\theta^4+24\theta^3+16\theta^2+6\theta+1) \tag{20}$$

4.3.2.3 Characteristics Summary

The KRSS filter exhibits the following characteristics:

a. The filter combination is recursive.

b. Starting transients will be essentially absent. That is, transients due to an incorrect a priori estimate of the state vector will be eliminated.

c. The algorithm is completely self-starting. After precisely 3 cycles of the second degree algorithm, the initial values will have been completely dropped.

d. At each cycling of the expanding memory filter, its error covariance matrix corresponds to a least-squares processing of all previous points. At each cycling of the fading memory filter, its error covariance matrix corresponds to an "exponentially weighted" least-squares fit on the data presented to it.

e. Dynamic response can be balanced off against random noise suppression during fading memory operation by

the appropriate choice of the filter control parameter θ. Setting θ close to unity is best from a variance reduction standpoint and worst from a dynamic response standpoint. The reduction of θ from unity improves dynamic response at the expense of variance reduction. This balance can be maintained throughout the entire tracking interval by appropriate adjustments to θ.

f. The expanding memory filter provides a method for changing in a continuous fashion the stress placed on the data. For example, a simple process for setting the stress value placed on reacquired raw data subsequent to a "data dropout" for a period of time can be devised so that the stress to be applied is determined by the length of the dropout interval and the initialization state at the time dropout occurred. This would involve decrementing the data cycle counter N by appropriate amounts during the dropout interval until data was again received.

4.3.3 Filter Operation

4.3.3.1 General

In the utilization of filtering algorithms, a compromise must be made between the conflicting requirements of good noise suppression (heavy filtering, sluggish system) and of responsiveness to changes (light filtering, quick reacting system). This compromise applies to both the expanding memory filter and the fading memory filter. For the fading memory filter with a fixed sample interval τ the choice of the filter control parameter θ will be the sole determinant of the fading memory filter response to noise and systematic changes in the input data such as transients in acceleration. For the expanding memory filter with a fixed sample interval τ the number of samples N will dictate

the relative weight given to (1) noise suppression and (2) filter responsiveness. Initially, for small values of N, the expanding memory filter will be extremely responsive, with correspondingly poor noise suppression characteristics. As N increases, filter responsiveness will decrease and noise suppression will improve. At a specific value of N, which depends on the fading memory filter control parameter θ (see Equation 15), the transition is made to the fading memory filter. Consequently, θ is the single parameter which can be studied to specify what this balance must be for a particular application for the expanding-fading memory filter combination.

4.3.3.2 Filter Performance Indices

Filtering errors are caused primarily by (1) filter response to noise on the input data and (2) the inherent dynamic lag of the filter. In order to properly assess these two independent error sources, performance indices have been adopted which depend only on the filter parameters θ and τ; where θ is the fading memory control parameter and τ is the input data interval.

4.3.3.2.1 Velocity Variance Reduction Factor

For noise suppression the generally accepted performance index is VRF. For range safety applications the "velocity" VRF (rather than position or acceleration) has been found to be of primary importance and is defined as follows:

$$VRFV(J) = \frac{\sigma_V^2}{\sigma_P^2}(J) \tag{21}$$

$$= \frac{\text{Variance in Velocity Estimate at Point "J"}}{\text{Variance in Raw Position Input Data}}$$

and has the units of $(1/sec)^2$.

The velocity estimated variance is dependent upon the point being evaluated. If the velocity variance is to be evaluated at the current point (J=0), its value will be lower than if it is to be evaluated for a 1-step prediction (J=1), since extrapolations beyond the data base produce variance increases.

For the fading memory filter this factor is a function of J, θ and τ denoted functionally by:

$$VRFV(J, \theta, \tau) = \frac{\sigma_V^2}{\sigma_P^2}(J, \theta, \tau) \tag{22}$$

to emphasize that this performance index is an explicit function of only these independent variables. However, in subsequent discussions, the simpler form (VRFV(J)) will be used, unless functional dependence is to be stressed.

For a 1-step prediction the value for VRFV(1) is given by Equation (17). For a current point evaluation for VRFV(0) Equation (18) should be used. This current point evaluation of the velocity variance reduction factor has been adopted as the performance index which characterizes the noise suppression capability of the KRSS filter.

Figure 5 shows both the noise suppression performance index chosen (VRFV(0) = VRFV(0, θ, τ)) and its associated 1-step prediction value (VRFV(1)), plotted against the filter control parameter θ. This functional relationship will be used subsequently in the development of the proper operating value for θ.

4.3.3.2.2 Filter Dynamic Lag

To establish an index of performance for filter

dynamic lag, a typical systematic change in the input data can be considered to be a step change in acceleration. In the absence of random and bias errors the following transient position data history may be assumed to be actual raw data for each component; the position data being the only information acted on by the filter.

	$t<t_A$	$t \geq t_A$	Input Change In
$\ddot{x}(t)$	0	A (constant)	Acceleration
$\dot{x}(t)$	0	$A\ (t-t_A)$	Velocity
$x(t)$	0	$A\ (t-t_A)^2/2$	Position

Where A is the acceleration transient step magnitude applied at time t_A. Emphasis should be placed here on the fact that A is a step or change in acceleration and should not be confused with the vehicle total acceleration.

For a particular value of A, a fixed data sample interval τ, and a choice of the control parameter θ, the steady state (initialized) fading memory filter can accept position data as input which is consistent with a step input in acceleration A, i.e., $x(t)=A(t-t_A)^2/2$, applied at $t=t_A$. The filter response to just this step transient will exhibit a velocity error (V_{LAG}) which will eventually damp out, as indicated by Figure 6. The time delay due to the filter ΔT_{FILTER} is defined as the time interval over which a constant acceleration A acts to produce the maximum velocity error observed, $(\Delta V_{LAG})_{MAX}$:

$$(\Delta V_{LAG})_{MAX} = A\ (\Delta T_{FILTER})$$

However, since the filter is linear, the filter time delay ΔT_{FILTER} is independent of the step magnitude A. Functionally we have:

$$\Delta T_{FILTER} = \Delta T_{FILTER}(J,\ \theta,\ \tau) \tag{23}$$

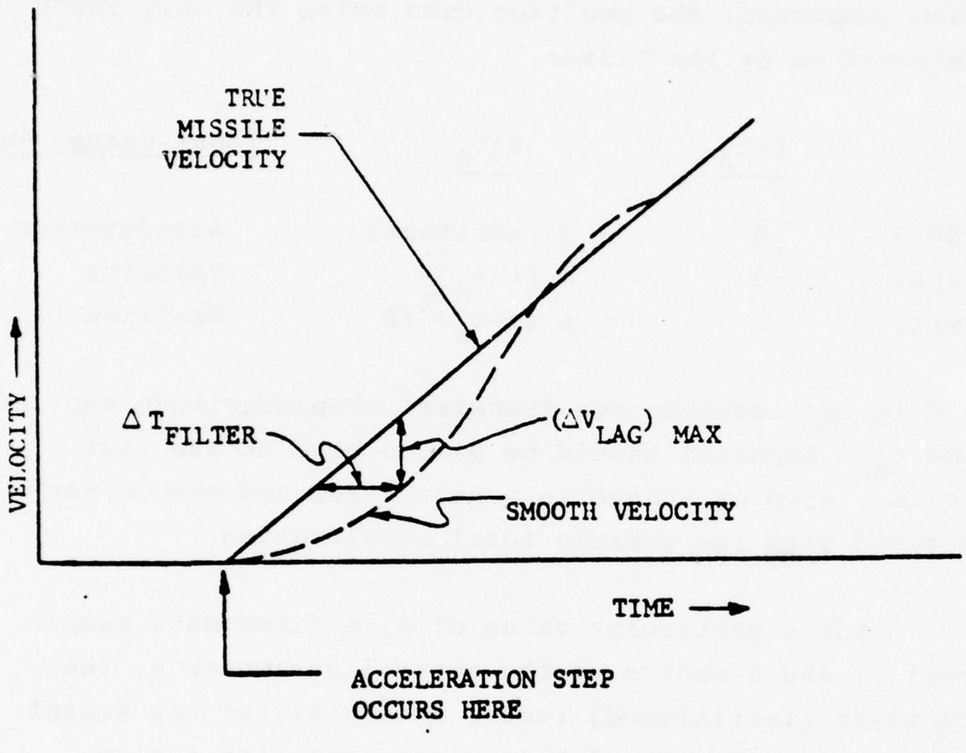


FIGURE 6. ILLUSTRATION DEPICTING ELEMENTS OF FILTER DYNAMIC LAG

The filter dynamic time lag (ΔT_{FILTER}) depends upon (1) the point where the evaluation is made (i.e., J=0 for current point, J=1 for a 1-step prediction evaluation), (2) the filter control parameter θ and (3) the sample interval τ. Therefore, we adopt the function

$$\Delta T_{FILTER}(0) = \Delta T_{FILTER}(0, \theta, \tau)$$

as the performance index which characterizes the dynamic lag response of the fading memory filter.

Figure 7 shows the dynamic time lag of the fading memory filter versus θ. There is a significant difference between the lags evaluated at the current point (J=0) or at a 1-step prediction, with the lower curve (current point) chosen as the performance index of KRSS filter dynamic lag.

4.3.3.3 Velocity Error Minimization

Defining performance indices in the above manner permits the calculation of a "combined" velocity error, ΔV_{FILTER}, as follows:

$$\Delta V_{FILTER} = \sqrt{A^2(\Delta T_{FILTER})^2 + (N\sigma_p)^2 \ VRFV} \tag{24}$$

where ΔT_{FILTER} and VRFV are the previously defined filter performance indices, and the system parameters

σ_p = Standard deviation of the input position data.

N = Number of standard deviations. This number indicates the weight which should be placed on the random position error in Equation (24). When the step acceleration value A used is considered to be a "maximum" value, a

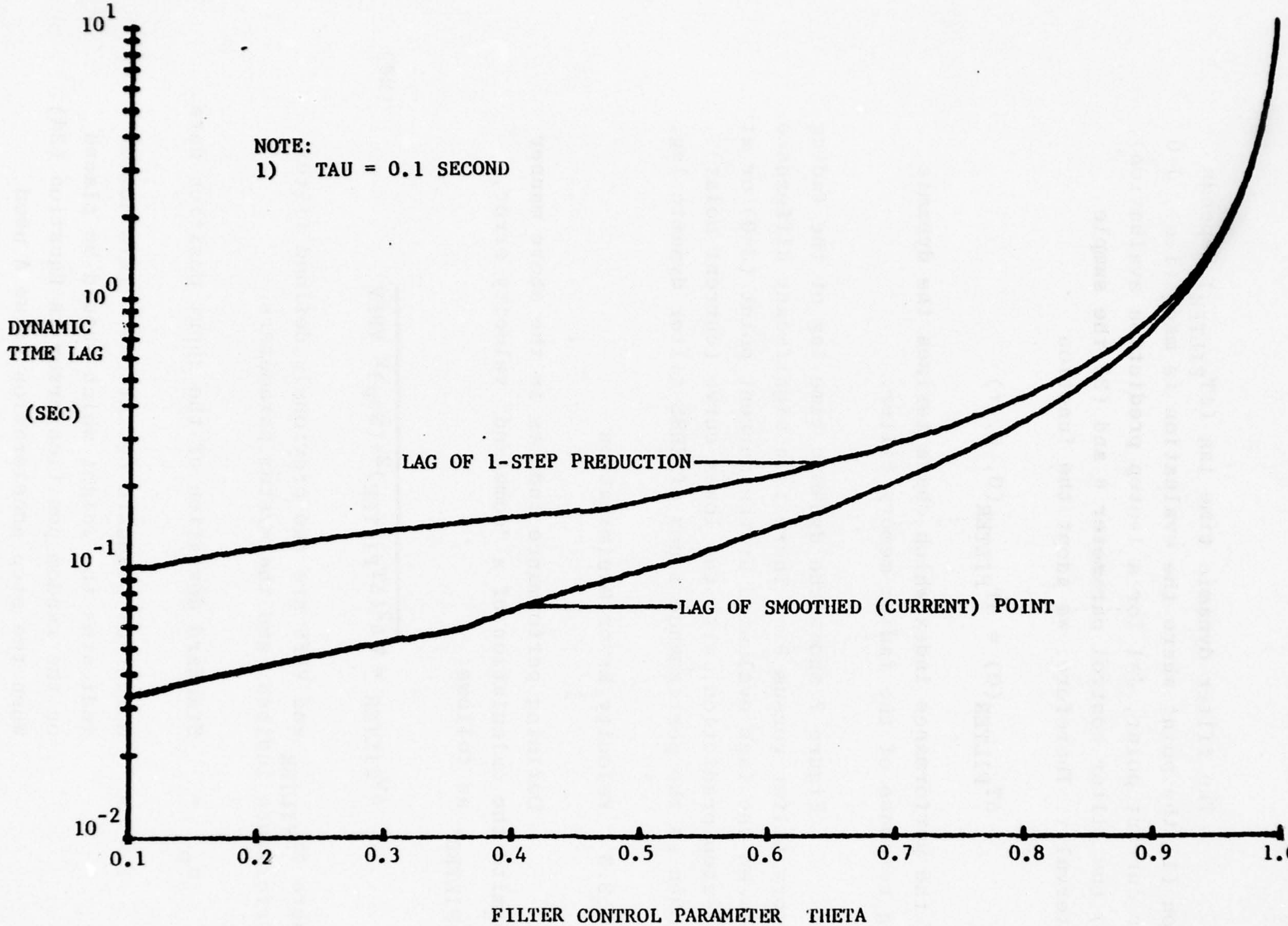


FIGURE 7. DYNAMIC TIME LAG OF FADING MEMORY FILTER VERSUS "THETA"

value of 3 for N is typically used. If the step acceleration value A is considered to be an "average" value, a value of 1 for N is used.

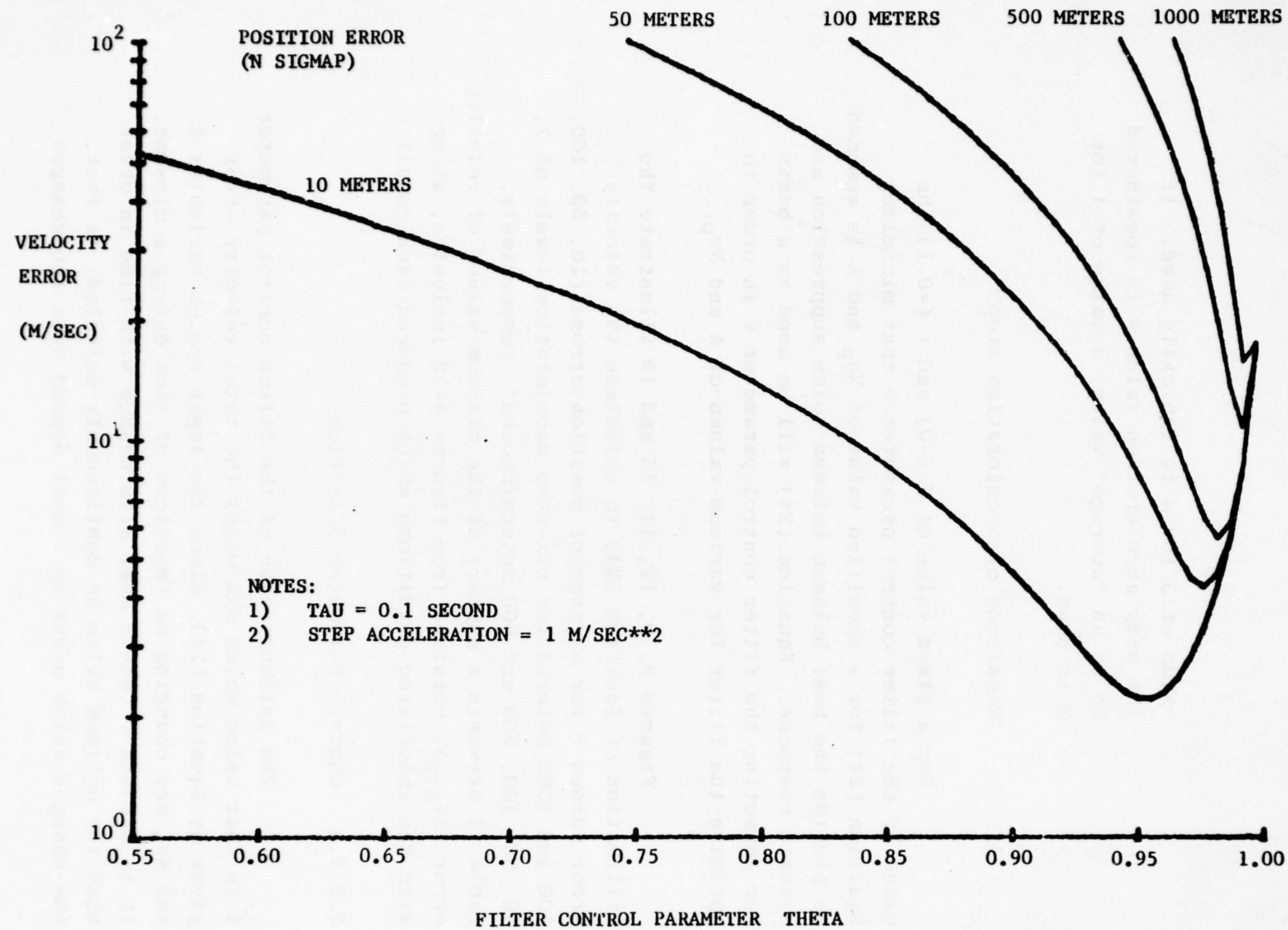
A = Magnitude of acceleration step.

For a fixed value of J (=0) and τ (=0.1) the value of the filter control parameter θ that minimizes Equation (24) for a specified value of $N\sigma_p$ and A is assumed to provide the best balance between noise suppression and dynamic response. Equation (24) will be used as a basis for selecting the filter control parameter θ in order to optimize the filter for various values of A and $N\sigma_p$.

Figures 8, 9, 10, 11, 12 and 13 illustrate the utilization of Equation (24) to calculate the velocity error versus θ for a range of position errors (10, 50, 100, 500 and 1000 meters) for six-step acceleration levels of 1, 10, 50, 100, 250 and 500 meters/second^2, respectively. Table II presents a summary of the minimum values of velocity error (ΔV_{MIN}), obtained from Figures 8-13 inclusive, along with the associated conditions which produced each result.

4.3.3.4 Control Parameter Selection

The optimum value of the filter control parameter θ is that value which minimizes the total velocity error given by Equation (24). Since the input system variables A and $N\sigma_p$ are changing as functions of time during a mission, it is necessary that θ must also change with time in order that the optimum value be continuously selected. In fact, the changes which occur to θ must depend upon the changes



FILTER 8. VELOCITY ERROR VERSUS "THETA"

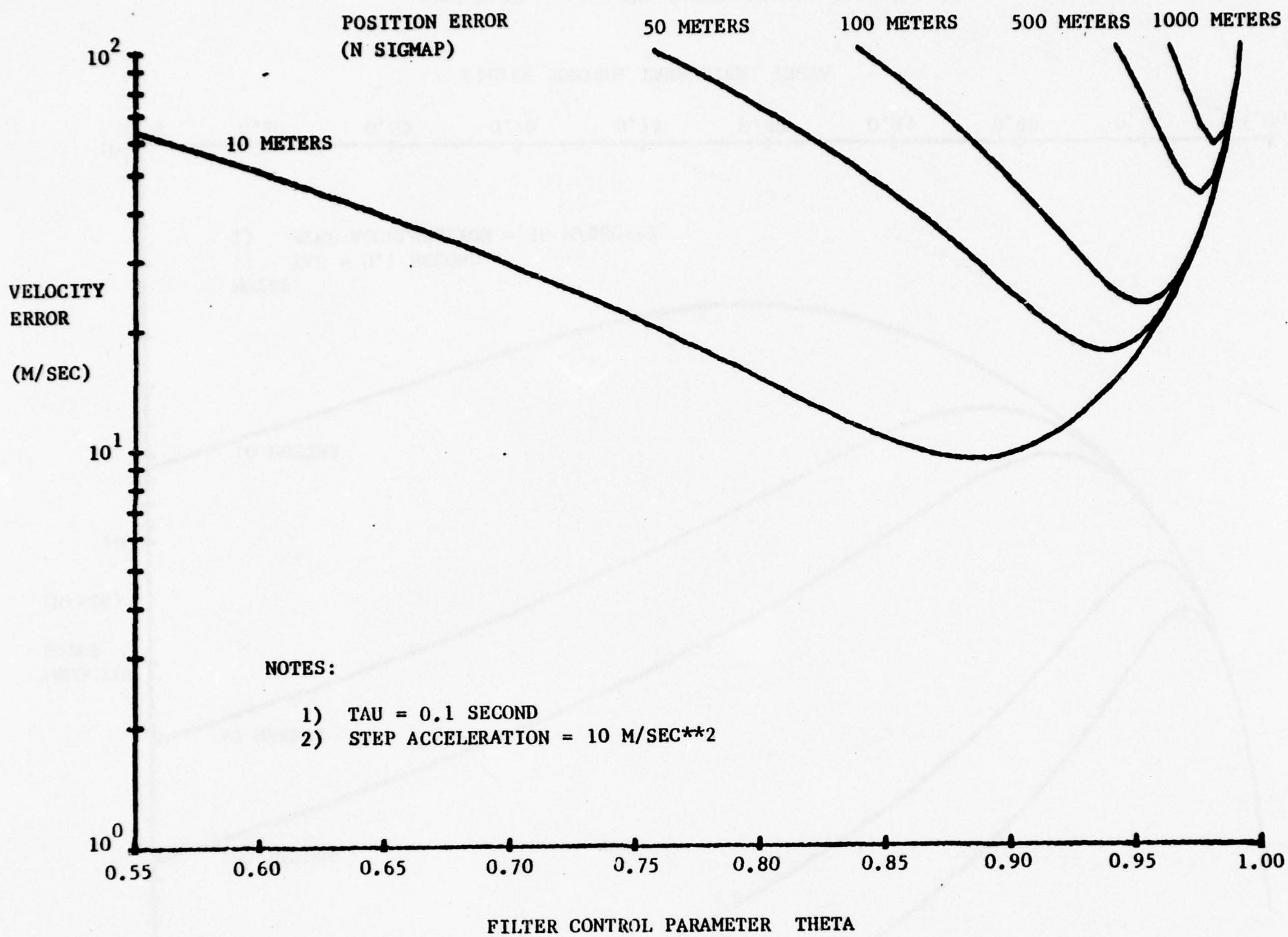


FIGURE 9. VELOCITY ERROR VERSUS "THETA"

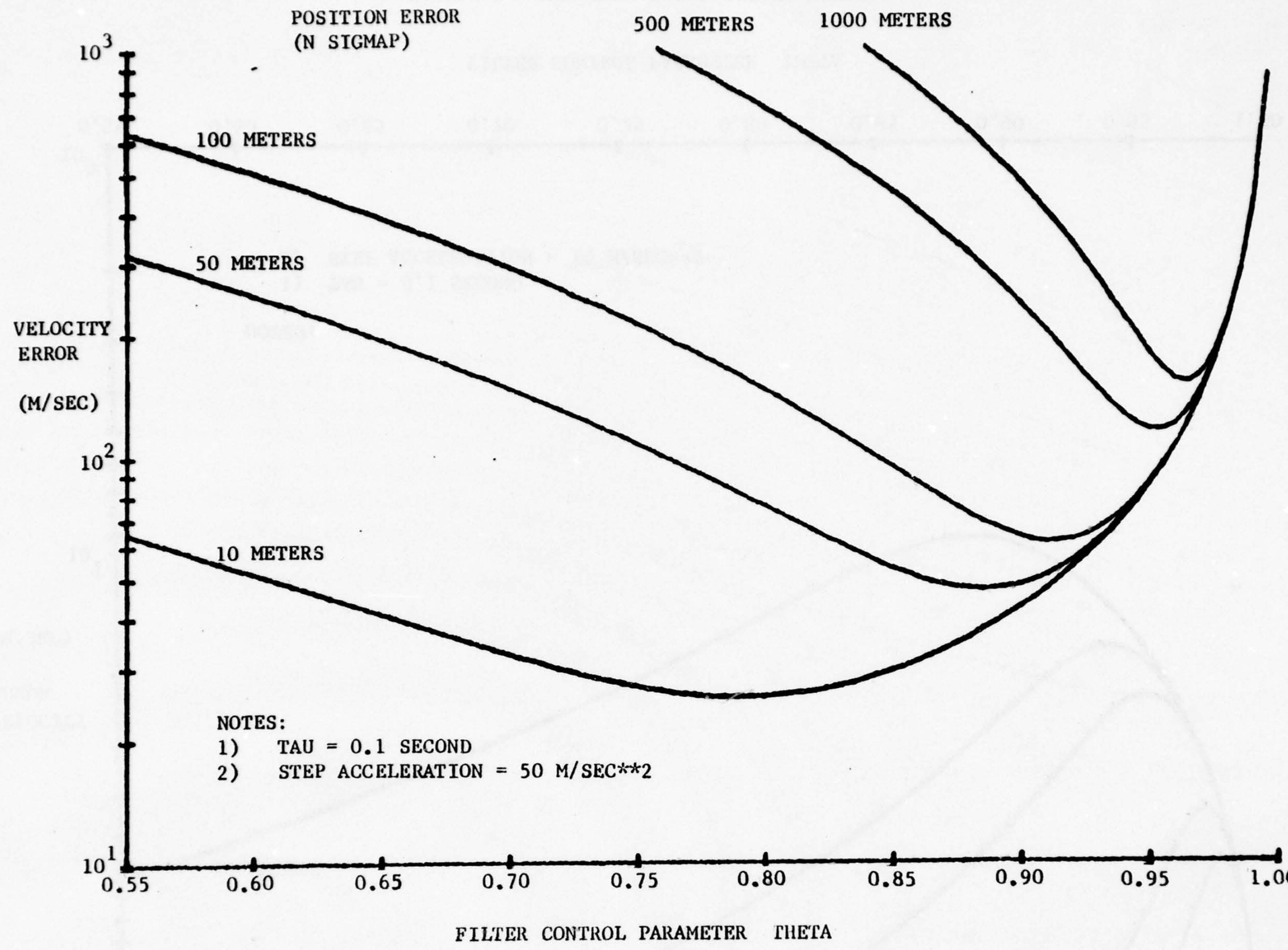


FIGURE 10. VELOCITY ERROR VERSUS THETA"

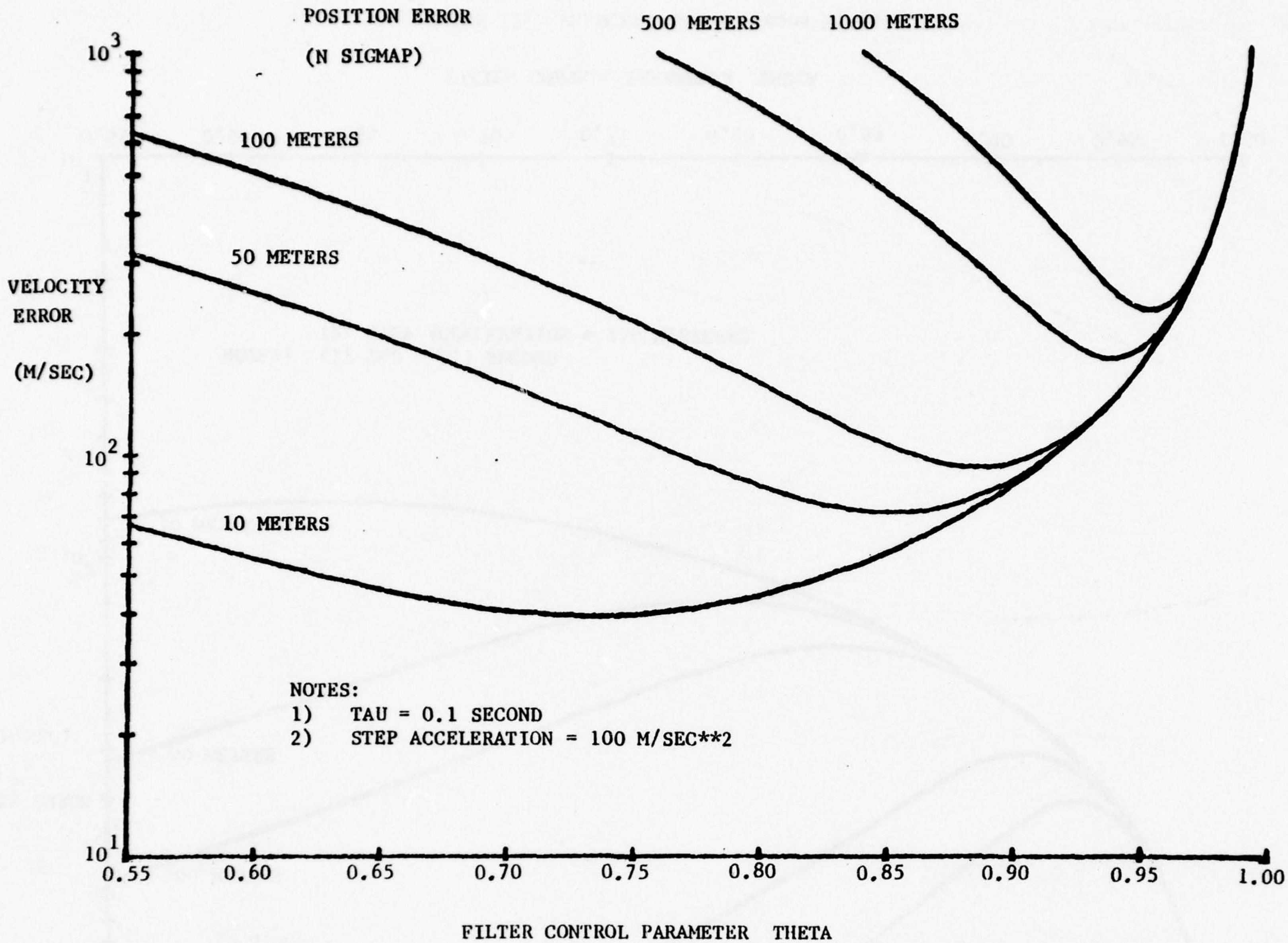


FIGURE 11. VELOCITY ERROR VERSUS "THETA"

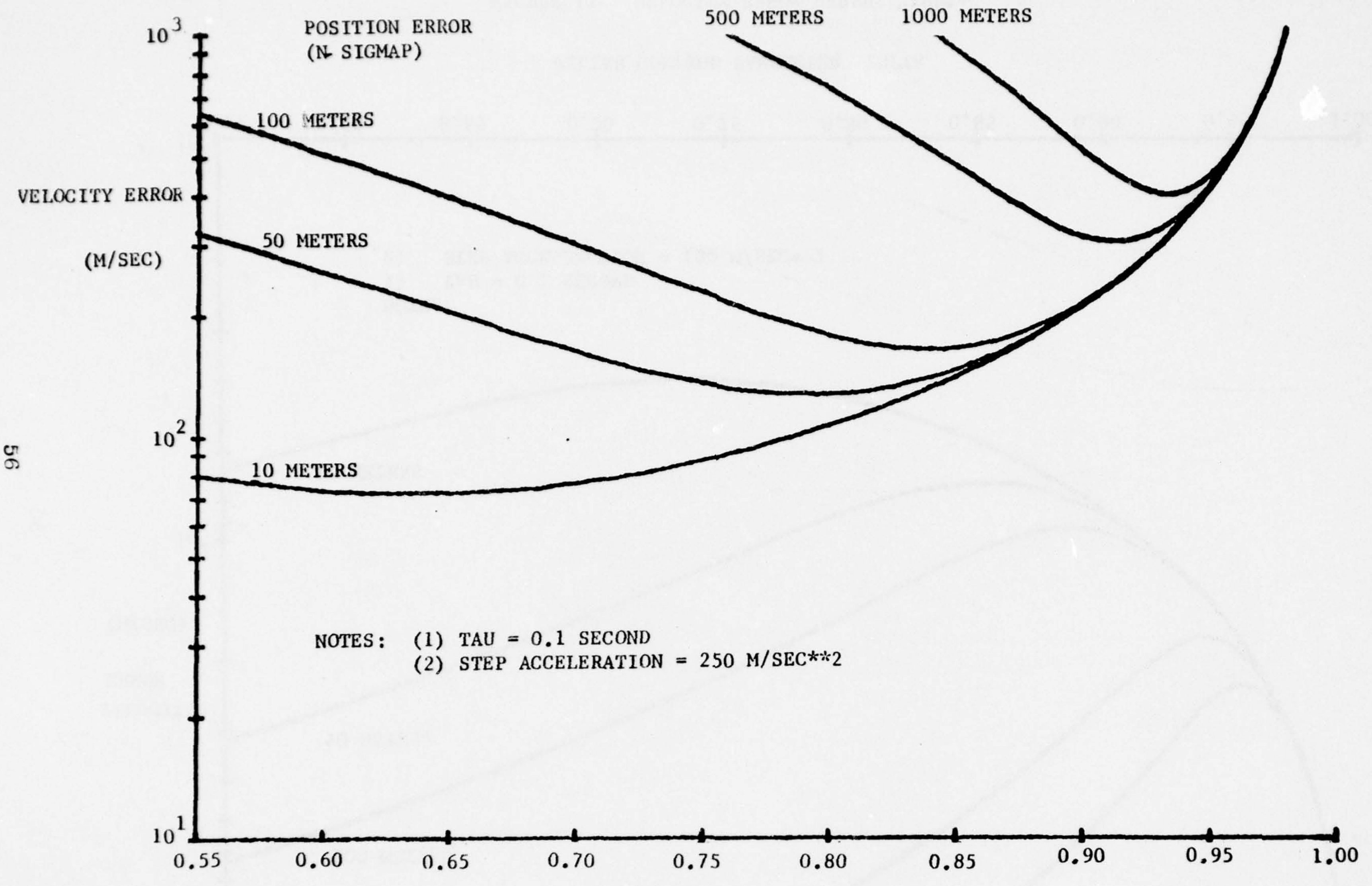


FIGURE 12. VELOCITY ERROR VERSUS "THETA"

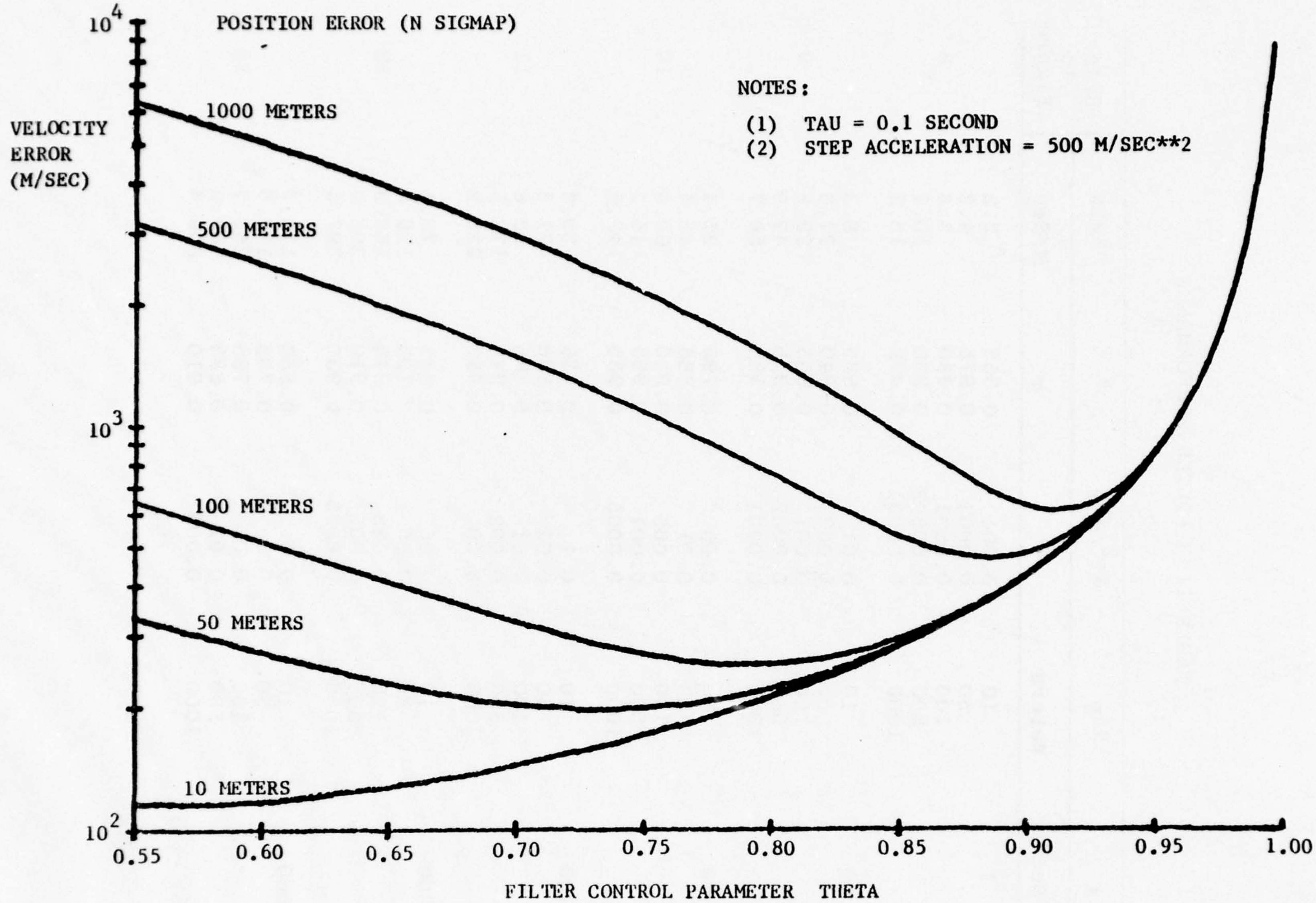


FIGURE 13. VELOCITY ERROR VERSUS "THETA"

TABLE II. FILTER PERFORMANCE

A	$N\sigma_p$	$A\tau^2/N\sigma_p$	θ	ΔV_{MIN}	Refer to Figure
M/Sec^2	Meters	-	-	M/Sec	
1	10	0.001	0.955	2.2	
	50	0.0002	0.975	4.2	
	100	0.0001	0.980	5.6	8
	500	0.00002	0.990	10.7	
	1000	0.00001	0.992	15.7	
10	10	0.01	0.885	9.3	
	50	0.002	0.940	17.3	
	100	0.001	0.955	22.8	9
	500	0.0002	0.975	42.9	
	1000	0.0001	0.980	56.7	
50	10	0.05	0.795	25.4	
	50	0.01	0.885	46.4	
	100	0.005	0.910	60.6	10
	500	0.001	0.955	114.1	
	1000	0.0005	0.965	149.6	
100	10	0.1	0.735	39.4	
	50	0.02	0.850	71.3	
	100	0.01	0.885	92.8	11
	500	0.002	0.940	173.5	
	1000	0.001	0.955	228.2	
250	10	0.25	0.635	72.1	
	50	0.05	0.795	126.8	
	100	0.025	0.840	163.9	12
	500	0.005	0.910	303.2	
	1000	0.0025	0.930	397.5	
500	10	0.5	0.550	115.7	
	50	0.1	0.735	197.0	
	100	0.05	0.795	253.7	13
	500	0.01	0.885	464.0	
	1000	0.005	0.910	606.4	

which occur to A and $N\sigma_p$. The dependence of θ on A and $N\sigma_p$ is clarified by rewriting Equation (24) in the following form:

$$\Delta V_{FILTER} = \left(\frac{N\sigma_p}{\tau}\right)\sqrt{\left[\tau^2 VRFV\right] + \left[\frac{A\tau^2}{N\sigma_p}\right]^2\left[\frac{\Delta T_{FILTER}}{\tau}\right]^2} \qquad (25)$$

The nondimensional parameter $[A\tau^2/N\sigma_p]$ has been found to characterize the effect of changes in A and $N\sigma_p$ on the minimization of ΔV_{FILTER}. This will be confirmed from the following discussion. The quantities $[\tau^2\ VRFV]$ and $[\Delta T_{FILTER}/\tau]$ can be considered to be dimensionless indices of performance, independent of τ, depending only on the filter control parameter θ. Equation (18) verifies that $[\tau^2 VRFV(0)]$ is indeed independent of τ. The dimensionless quantity $[\Delta T_{FILTER}/\tau]$ indicates the number of data cycles of dynamic lag resulting from a "normalized" step acceleration A_τ^2. The units of the quantity $(N\sigma_p)$ are the units of velocity, i.e., length/sec. Consequently, each term under the radical in Equation (25) is dimensionless, and independent of τ. A single valued functional relationship exists between θ and $(A\tau^2/N\sigma_p)$. That is, given the "input" quantity $(A\tau^2/N\sigma_p)$, there is one unique value of θ to minimize Equation (25), and that value has been defined as the optimum operating point for the fading memory filter. This relationship of θ versus $[A\tau^2/N\sigma_p]$ has been derived for the fading memory filter and is shown in Figure 14. The relationship given in this figure is keyed to the results of Table II.

The effect of θ on noise response and filter dynamic lag is presented in Figures 5 and 7, respectively. Using Equation (24) and the relationships represented by Figures 5 and 7, the performance curves of Figures 8 through

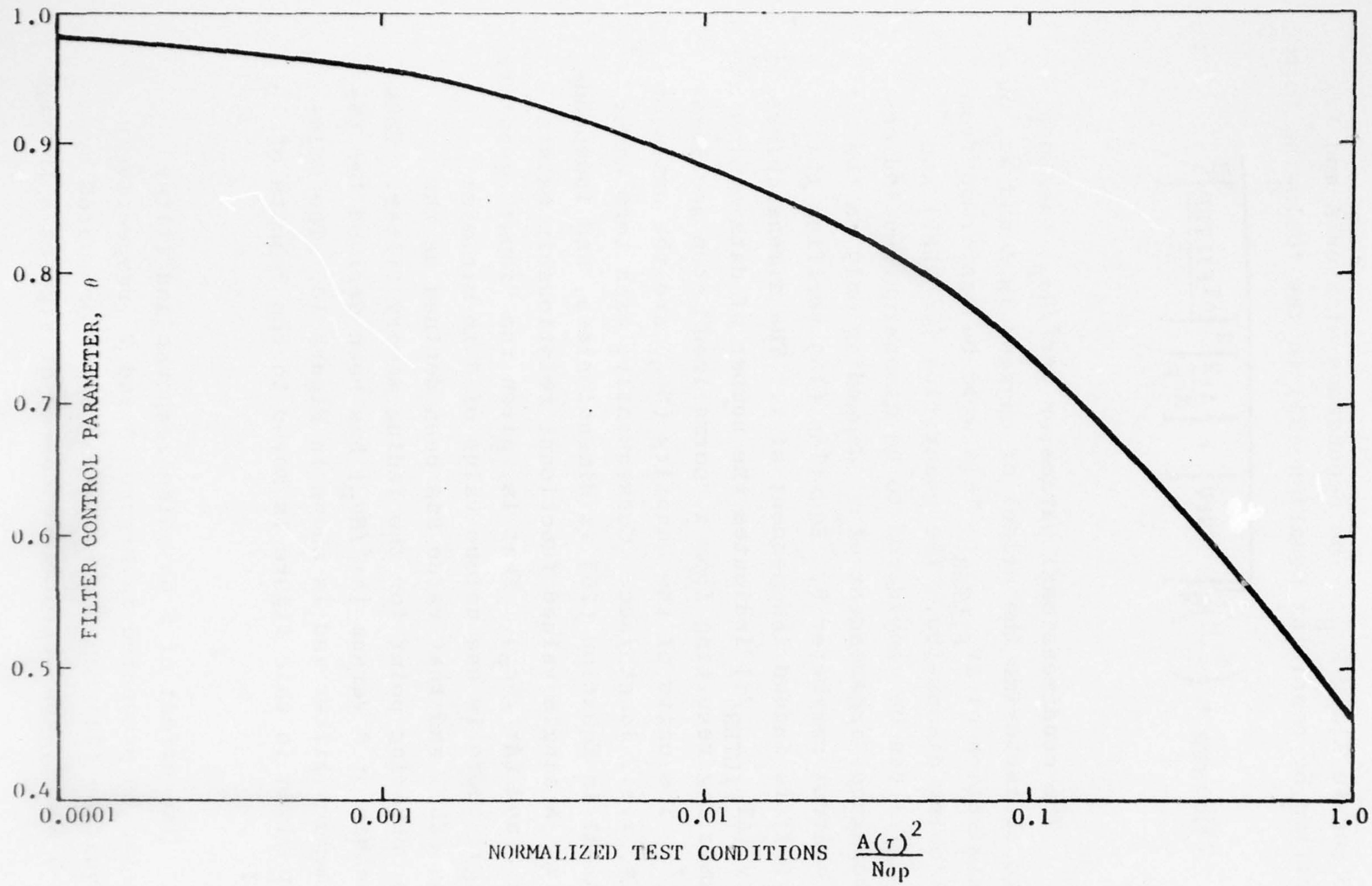


FIGURE 14. FILTER CONTROL PARAMETER, θ VS NORMALIZED TEST CONDITIONS $\frac{A(\tau)^2}{N\sigma p}$

13 were obtained. For each case there is a unique value of θ which minimizes the velocity error of the filter. These optimum θ values were plotted against normalized test conditions in Figure 14.

Figure 14 shows the relationship whereby optimum filtering can be maintained continuously during a mission provided detailed information on the time (or range) dependence of the acceleration transient A and input position noise error $N\sigma_p$ are known.

Figure 14 shows that optimum filter performance can be achieved if good estimates of A and $N\sigma_p$ can be determined. This translates filter performance parameters into quantities which are more physically related to the tracking problem. Generally, σ_p can be estimated as a function of range. The value of A for various phases of flight can also be determined for a particular mission. The choice of N, the weight applied to the position error σ_p, is a constant, subject to engineering judgment. When the step acceleration value A utilized is considered to be a "maximum" value, then the corresponding value of 3 for N is typically chosen. If the step acceleration value A is an "average" value, a value of 1 for N is selected.

4.4 PACIFIC MISSILE TEST CENTER

4.4.1 Exponential Filter

4.4.1.1 Development and Usage

The exponential filter as used at the Pacific Missile Test Center (PMTC) was originally developed for use on ICBM/IRBM class vehicle launches. It has been used subsequently on launches of shorter range surface-to-surface and air-to-surface missiles. The filter is used two ways. It is used on raw range azimuth and elevation data from each radar to provide expected values of the next observation for use in editing. The mean absolute difference between the expected and observed values is used to estimate the standard deviation of the noise as a basis for editing the raw data. The noise estimates from two or more radars are then compared to determine which has the least crossrange Instantaneous Impact Prediction (IIP) error. This radar data is then selected for further use.

The unsmoothed but edited data from the selected radar is converted to a Cartesian coordinate system. A second exponential filter operates on this data to obtain smoothed missile position and velocity.

4.4.1.2 Theory and Method of Operation

The theory of exponential smoothing assumes that the impact function can be represented by a second degree polynomial

$$X_t = a_o + a_1 t + \frac{a_2}{2} t^2.$$

A future value can be predicted by

$$\hat{X}_{t+z} = \hat{X}_t + Z \hat{\dot{X}}_t + \frac{z^2}{2} \hat{\ddot{X}}_t.$$

If a constant (zeroth degree) model is assumed, a recursive average may be obtained by

$$\hat{X}_t = \hat{X}_{t-1} + \alpha(X_t - \hat{X}_{t-1}),$$

where α is a smoothing constant, $0<\alpha<1$.

Let $\qquad B = 1 - \alpha$

and $\qquad S_t^1 = \hat{X}_t$

so that $\qquad S_t^1 = B\ S_{t-1}^1 + \alpha X_t.$

S_t^1 is referred to as the first order smoothing operator. A second order operator, S_t^2, may be defined by

$$S_t^2 = B\ S_{t-1}^2 + \alpha\ S_t^1.$$

Note that the superscript refers to the order of the operator and is not an exponent. A third order operator, S_t^3, is similarly defined.

It may be shown that linear combinations of these three operators can be used to give least-squares estimates of the coefficients of the polynomial.

$$A_o = 3S_t^1 - 3S_t^2 + S_t^3$$

$$A_1 = \frac{\alpha}{2B^2} [(6-5\alpha)\ S_t^1 - 2(5-4\alpha)S_t^2 + (4-3\alpha)S_t^3]$$

$$A_2 = \frac{\alpha^2}{2B^2} (S_t^1 - 2S_t^2 + S_t^3)$$

It may be seen then, that

$$X = A_o$$

$$\dot{X} = A_1/\Delta t$$

$$\ddot{X} = A_2/\Delta t^2$$

4.4.1.3 Outstanding Features

This filter provides a simple and efficient means to fit data to a second degree polynomial. It is flexible in that the value of α may be selected to give the desired compromise between response and smoothness.

4.4.1.4 Limitations

The filter is general and assumes nothing other than the fit to a second degree polynomial. Additional smoothness or response can be obtained by more specialized filters.

4.4.2 Recursive Exponentially Weighted Least-Squares Filter

4.4.2.1 Usage

The filter is exponential and acts as a one-step predictor. It is an expanding memory filter until the number of cycles reach the filter size number being used at that time. The filter switches from an expanding memory to a fading memory when the number of cycles is equal to or greater than the filter size. The telemetered vehicle's angular rate information is used to control the filter size

used during powered flight. For example, if a bad angular rate signal is received, then the short filter size is used, making the filter output noisier, but more responsive than when the long filter size is used.

4.4.2.2 Theory and Method of Operation

The theory of exponential smoothing assumes that the impact function can be represented by a second degree polynomial.

$$X_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2/2$$

A future value can be predicted by

$$P_{t+z} = P_t + ZV_t + Z^2 A_t/2$$

The equations used are obtained by filtering a second degree polynomial in time to the position, velocity, and acceleration data using a recursive exponentially weighted least-squares procedure. The equations are:

$$\Delta P_i = O_i - A_{i-1}$$

$$A_i = [A_{i-1} + \gamma_i \Delta P_i]K$$

$$Q_i = F_i C + A_i$$

$$V_i = V_{i-1} + 2Q_i + \beta_i \Delta P_i$$

$$P_i = P_{i-1} + V_i - Q_i + \alpha_i \Delta P_i$$

where

Q_i = Observed edited Cartesian coordinate at time t_i.

P_i = Smooth estimate of position for time t_i obtained from data taken up to and including that at time t_{i-1}.

V_i = Smooth estimate of velocity for time t_i obtained from data taken up to and including that at time t_{i-1}; the first time derivative of position.

A_i = Smooth estimate of acceleration reflecting the thrust of the vehicle for time t_i obtained from data taken up to and including that at time t_{i-1}, the second time derivative of position.

F_i = Smooth estimate of acceleration of the vehicle due to earth's gravity for time t_i obtained from data taken up to and including that at time t_{i-1}.

C = Conversion factor to make the acceleration-due-to-gravity term dimensionless

K = Forcing term used to minimize filter overshoot during the main engine cutoff (MECO) phase of the flight; this is a parameter entry whose value is approximately unity.

and

$$\alpha_J = 3(3J^2 + 3J + 2)/(J + 3)(J + 2)(J + 1)$$

$$\beta_J = 18(2J + 1)/(J + 3)(J + 2)(J + 1)$$

$$\gamma_J = 30/(J + 3)(J + 2)(J + 1)$$

where

α, β and γ are the weight coefficients where $J = 0,1,2,\ldots,N$. N is the predetermined filter size entered as a parameter before the operation. N may have two different values during powered flight; one value signifying a long-filter length for good-quality telemetry data, a second value signifying a short-filter length for bad-quality telemetry data and a third value (usually extremely large) for use during ballistic flight.

The following coefficients are used during ballistic flight:

$$\alpha_J = 2(2J + 1)/(J + 2)(J + 1)$$

$$\beta_J = 6/(J + 2)(J + 1)$$

The coefficient γ is not used since the thrust is zero.

4.4.2.3 Outstanding Features

The position data are rotated into an orientation such that the filter efficiency is maximized by minimizing the random and systematic radar tracking errors. This orientation is in the theoretical direction of the vehicle at MECO from the radar.

The filter takes advantage of the telemetry information during flight and utilizes the time of MECO, which is determinable. The quality of the signal is determined by the values of the pitch and yaw rates. The filter may also minimize overshoot error by eliminating residual thrust after burnout is sensed.

4.4.2.4 Limitations

This filter is designed to be used for vehicles going through a fairly continuous powered flight which, after becoming ballistic, remains so throughout the remainder of flight. With modifications, however, the capability of a reignition stage with powered flight followed by ballistic flight can be incorporated. Much analysis must be done on the vehicle in flight so that the many parameters can be computed. This filter is not as generalized as the one previously described, but it may present a more true estimation of the vehicle behavior.

The filter does need the telemetry data in conjunction with the radar data to be smoothed. Certain parameters are necessary, such as the orientation angles, for error minimization and approximate time of burnout as well as vernier engine thrust.

4.5 WALLOPS ISLAND

4.5.1 Free-Flight Filters

These filters were designed and reported by the Special Information Products Department (SIPD), Defense Electronics Division, General Electric Co., Syracuse, New York. They are an integral part of the Instantaneous Impact Prediction System supplied by General Electric in 1965 under contract NAS 6-852. General Electric designed the filters around Wallops radar data. Since its development minor changes have been made, but the basic filter has been preserved. These filters utilize the method developed by Rudolf Kalman. These filters are, however, special adaptations of the Kalman method in order to meet real-time requirements. For a detailed explanation of this technique refer to references 1 through 6 of Section 5.0.

4.5.1.1 This filter was designed and is used for extended periods of free flight or vehicle coast.

4.5.1.2 We define the state vector of the system to be:

$$\underline{X} \quad \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \qquad (1)$$

where each element is in the inertial coordinate frame.

In order to predict the state from one data sample to the next, we use the matrix equation

$$\underline{X}(n/n-1) = \underline{\phi}(n,n-1)\ \underline{X}\ (n-1/n-1) + \underline{T}\ \underline{W}\ (n-1) \qquad (2)$$

where $\underline{W}$ is the gravity model which is covered in subparagraph 4.5.3. The notation A(n/m) will be used to designate an estimation of A at time n based on observations through time m.

For our system we know the following scalar equations:

$$\left.\begin{aligned} \underline{X}(n/n-1) &= x(n-1/n-1) + t\dot{x}(n=1/n-1) + \frac{\tau^2}{2}\ \ddot{x}\ (n-1) \\ \dot{x}(n/n-1) &= \dot{x}(n-1/n-1)+t\ddot{x}(n-1) \end{aligned}\right\} \qquad (3)$$

where t is the time difference between data samples and x denotes a time derivative. Similar equations can be written for y and z. These scalar equations can be put into the matrix form of Equation (2) to yield the ϕ and T matrices.

$$\underset{\underline{X}(n/n-1)}{\underset{\downarrow}{\begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}_{(n/n-1)}}} = \underset{\underline{\phi}(n,n-1)}{\underset{\downarrow}{\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & t & 0 & 0 \\ 0 & 1 & 0 & 0 & t & 0 \\ 0 & 0 & 1 & 0 & 0 & t \\ \hdashline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]}} \underset{\underline{X}(n-1/n-1)}{\underset{\downarrow}{\begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}_{(n-1/n-1)}}} + \underset{\underline{T}}{\underset{\downarrow}{\left[\begin{array}{ccc} \frac{t^2}{2} & 0 & 0 \\ 0 & \frac{t^2}{2} & 0 \\ 0 & 0 & \frac{t^2}{0} \\ \hdashline t & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & t \end{array}\right]}} \underset{\underline{W}(n-1)}{\underset{\downarrow}{\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix}_{(n-1)}}} \qquad (4)$$

The state vector is related to the measured data by the equation

$$\underline{Z}(t) = \underline{HX}(t) + \underline{V}(t) \qquad (5)$$

where $\underline{Z}(t)$ is the measurement, $\underline{H}$ is the transformation matrix and $\underline{V}(t)$ is the measurement error vector. Since we are measuring range (R), azimuth (A) and elevation (E), this equation is:

$$\begin{bmatrix} R \\ A \\ E \end{bmatrix} = \underline{H} \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} \epsilon_R \\ \epsilon_A \\ \epsilon_E \end{bmatrix} \qquad (6)$$

Since our state vector is in the inertial frame, we must transform from there to fixed earth, to radar Cartesian and finally to radar coordinates.

Let Φ_g = Radar longitude from Greenwich

t_p = Program time

r = Earth's rotation rate

$\Phi = \Phi_g + rt_p$

Then we have

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix}_{(n/n-1)} = \underline{A}^T \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_I \\ y_I \\ z_I \end{bmatrix}_{(n/n-1)} - \underline{A}^T \begin{bmatrix} x_e \\ 0 \\ z_e \end{bmatrix} \tag{7}$$

This transformation, including a derivation of A, is discussed in detail in subparagraph 4.5.4.

Since

$$R(n/n-1) = [u^2(n/n-1) + v^2(n/n-1) + w^2(n/n-1)]^{\frac{1}{2}}$$

$$A(n/n-1) = \tan^{-1} \left[\frac{u(n/n-1)}{v(n/n-1)}\right] \tag{8}$$

$$E(n/n-1) = \tan^{-1} \left[\frac{w(n/n-1)}{\rho}\right]$$

$$\text{where } \rho = [u^2(n/n-1) + v^2(n/n-1)]^{\frac{1}{2}}$$

Since

$$\underline{R}u = \begin{bmatrix} \frac{dR}{du} & \frac{dR}{dv} & \frac{dR}{dw} \\ \frac{dA}{du} & \frac{dA}{dv} & \frac{dA}{dw} \\ \frac{dE}{du} & \frac{dE}{dv} & \frac{dE}{dw} \end{bmatrix} \tag{9}$$

This becomes

$$\underline{K}u = \begin{bmatrix} \frac{u}{R} & \frac{v}{R} & \frac{w}{R} \\ \frac{v}{\rho^2} & \frac{-u}{\rho^2} & 0 \\ \frac{-uw}{R^2\rho} & \frac{-vw}{R^2\rho} & \frac{\rho}{R^2} \end{bmatrix} \tag{10}$$

Let

$$\underline{h} = \underline{K}u\, A^T \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

and

$$\underline{H} = [\underline{h} \;\; \underline{0}] \qquad \text{where } \underline{0} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{12}$$

We now formulate the filter equations. The filter consists of the following three equations:

$$\underline{X}(n/n-1) = \underline{X}(n/n-1) + \underline{K}(n)\, \Delta\underline{Z}(n) \tag{13}$$

$$\underline{K}(n) = \underline{P}(n/n-1)\underline{H}^T\, [\underline{HP}(n/n-1)\underline{H}^T + S_{-Z}]^{-1} \tag{14}$$

$$\underline{P}(n/n-1) = \phi\{\underline{P}^-\underline{H}^T\, [\underline{HP}^-H^T + S_{-Z}]^{-1}\underline{HP}^-\}\phi + \underline{T}\,\underline{Q}\,\underline{T}^T \tag{15}$$

where

$\underline{P}$ = Covariance matrix of the state vector

$\underline{P}^-$ = P(n-1/n-2)

$\Delta\underline{Z}(n)$ = Difference between measured (R,A,E) and predicted (R,A,E)

$\underline{S}_z$ = Measurement covariance matrix

$\underline{K}(n)$ = Weighting matrix

$\underline{Q}$ = Covariance of W (the gravity model) which we assume to be zero (i.e., perfect gravity model).

Equation (15) reduces to

$$\underline{P}(n/n\text{-}1) = \underline{\Phi P}^- \underline{\Phi}^T - \underline{\Phi P}^- \underline{H}^T [\underline{H P}^- \underline{H}^T + \underline{S}_z]^{-1} \underline{H P \Phi}^T$$

or

$$\underline{P}(n/n\text{-}1) = \underline{\Phi P}^- \underline{\Phi}^T - \underline{\Phi K}(n) \underline{HP}^- \underline{\Phi}^T \tag{16}$$

Now define

$$\underline{M}_u = [\underline{H P}^- \underline{H}^T + \underline{S}_z]^{-1} \tag{17}$$

Then

$$\underline{K}(n) = \underline{P}^- \underline{H}^T \underline{M}_u \tag{18}$$

Now, referring to Equation (13), we have the best estimate of the state given all data up to "now":

$$\underline{Z}(n/n) = \underline{X}(n/n-1) + \underline{K}(n) \begin{bmatrix} R(n) - R(n/n-1) \\ A(n) - A(n/n-1) \\ E(n) - E(n/n-1) \end{bmatrix} \tag{19}$$

Where $\underline{X}(n/n-1)$ is from Equation (4).

R(n), A(n), E(n) are corrected rate measurements now.

R(n/n-1), A(n/n-1), E(n/n-1) are from Equation (8) and $\underline{K}(n)$ is from Equation (18).

As it is used by NASA at Wallops Island, Virginia, the free-flight filter depends on the powered-flight filter (subparagraph 4.5.2) for initialization of the state vector. An initial estimate of the state vector is computed from the output of the last cycle of the powered-flight filter. The covariance matrix is initialized as follows:

$$\underline{P} = \begin{bmatrix} 10^6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10^6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10^4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10^4 \end{bmatrix}$$

4.5.1.3 Outstanding Features

a. Both the free-flight and the powered-flight filters are end-point filters, which is essential for real-time applications.

b. Both filters are recursive; thus, only one data sample must be stored in the computer at any one time.

4.5.1.4 Limitations

Both the free-flight and the powered-flight filters are sensitive to the data editing scheme.

4.5.2 Powered-Flight Filter

4.5.2.1 This is the primary range safety filter since it is used during the powered portion of flight and for short periods of coast.

As burnout is approached during a powered stage the uncertainty of the acceleration terms in the powered flight model is increased. This is done by setting the variance of $\ddot{R}$, $\ddot{A}$ and $\ddot{E}$ to a relatively large value. This action enables the filter to "turn the corner" in its calculation of velocity. The uncertainty is kept large until after the next stage has fired. Then the filter covariance matrix is again allowed to converge until the burnout of this stage is approached. The reason for the covariance being kept large during free flight is the immediate response of the filter to accelerations during this period. This may be due to programmed or premature ignition.

NOTE: For the 5-second interval from T-2.5 sec. to T+2.5 sec., where T is a nominal staging time,

$$P_{R_{33}} = 1000. \times P_{R_{33}}$$

$$P_{A_{33}} = 100. \times P_{A_{33}}$$

$$P_{E_{33}} = 100. \times P_{E_{33}}$$

where P_R, P_A and P_E are the covariance matrices for range, azimuth and elevation, respectively. For example,

$$P_R = \begin{bmatrix} \sigma^2 R & \sigma_{R\dot{R}} & \sigma_{R\ddot{R}} & \sigma_{R\dddot{R}} & \sigma_{R\epsilon_R} \\ \sigma_{RR} & \sigma^2 \dot{R} & & & \\ & & \sigma^2 \ddot{R} & & \\ & & & \sigma^2 \dddot{R} & \\ & & & & \sigma^2 \epsilon_R \end{bmatrix} \begin{array}{l} \\ \\ \longrightarrow \sigma^2 \ddot{R} = P_{R_{33}} \\ = \text{variance of } R \\ \\ \end{array}$$

4.5.2.2 This recursive filter uses the third degree polynomial.

$$R = R_o + \dot{R}t + \ddot{R}\frac{t^2}{2} + \dddot{R}\frac{t^3}{6} \tag{20}$$

where t is time and the "dot" stands for time differentiation and the second degree polynomials

$$A = A_o + \dot{A}t + \ddot{A}\frac{t^2}{2} \tag{21}$$

$$E = E_o + \dot{E}t + \ddot{E}\frac{t}{2} \tag{22}$$

for azimuth and elevation models, respectively. To prevent a bias in the outputs of the filters, data is weighted out in exponential fashion and an error term is introduced into the model. The state vectors then become

$$\underline{R} = \begin{bmatrix} R \\ \dot{R} \\ \ddot{R} \\ \dddot{R} \\ \epsilon_R \end{bmatrix}, \underline{A} = \begin{bmatrix} A \\ \dot{A} \\ \ddot{A} \\ \epsilon_A \end{bmatrix}, \underline{E} = \begin{bmatrix} E \\ \dot{E} \\ \ddot{E} \\ \epsilon_E \end{bmatrix} \tag{23}$$

where ϵ_R, ϵ_A and ϵ_E are the error terms in range, azimuth and elevation, respectively.

The following will apply only to range, but similar arguments apply to azimuth and elevation. We would like to write Equation (20) in the form of the matrix equation

$$\underline{R}(n/n-1) = \underline{\phi}(n, n-1)\underline{R}(n-1/n-1) + \underline{W}(n) \tag{24}$$

where the notation A(n/m) refers to an estimate of some quantity A at a time n based on observations up to and including time m. The $\underline{W}(n)$ is the system forcing function which we assume to be zero with a covariance of zero. We can write:

$$\underset{\underline{R}(n/n-1)}{\begin{bmatrix} R \\ \dot{R} \\ \ddot{R} \\ \dddot{R} \\ \epsilon_R \end{bmatrix}_{(n/n-1)}} = \underset{\underline{\phi}}{\begin{bmatrix} 1 & t & \frac{t^2}{2} & \frac{t^3}{6} & \frac{t^3}{6} \\ 0 & 1 & t & \frac{t^2}{2} & \frac{t^2}{2} \\ 0 & 0 & 1 & t & t \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & \alpha_R \end{bmatrix}} \underset{\underline{R}(n-1/n-1)}{\begin{bmatrix} R \\ \dot{R} \\ \ddot{R} \\ \dddot{R} \\ \epsilon_R \end{bmatrix}_{(n-1/n-1)}} \underset{\underline{W}}{+\underline{0}} \tag{25}$$

where α_R is a constant (presently equal to 0.5). The state vector is related to the measured data by the equation

$$\underline{Z}(t) = \underline{HX}(t) + \underline{V}(t) \tag{26}$$

where $\underline{Z}(t)$ is the measurement, $\underline{H}$ is the transformation matrix and V(t) is the measurement error vector. Since we are measuring range, we have

$$R = [1\ 0\ 0\ 0\ 0] \begin{bmatrix} R \\ \dot{R} \\ \ddot{R} \\ \dddot{R} \\ \epsilon_R \end{bmatrix} + V(t). \tag{27}$$

So $\underline{H} = [1\ 0\ 0\ 0\ 0]$.

The filter consists of the following equations:

$$\underline{R}(n/n) = \underline{R}(n/n-1) + \underline{K}(n)\,[\underline{R}(n) - \underline{R}(n/n-1)] \tag{28}$$

$$\underline{K}(n) = \underline{P}(n/n-1)\,\underline{H}^T\,[\underline{H}\underline{P}(n/n-1)\,\underline{H}^T + \underline{S}_R]^{-1} \tag{29}$$

$$\underline{P}(n/n-1) = \underline{\phi}\{\underline{P}^- - \underline{P}^-\,\underline{H}^T\,[\underline{H}\underline{P}\,\underline{H}^T + \underline{S}_R]^{-1}\underline{H}\underline{P}^-\}\underline{\phi} \tag{30}$$

where

$\underline{P}$ = Covariance matrix for range state vector

$\underline{P}^-$ = $P(n-1/n-2)$

$\underline{R}(n)$ = Present range measurement

$\underline{S}_R$ = Variance of measured range

$\underline{K}(n)$ = Weighting matrix

$\underline{R}(n/N-1)$ is computed from Equation (25)

The filter is initialized as follows:

$$\underline{P}(1/0) = \begin{bmatrix} 10S_R & 0 & 0 & 0 & 0 \\ 0 & 10{,}000 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 1000 \end{bmatrix}$$

$$\underline{R}(O) = \begin{bmatrix} R_O \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (31)$$

where R_O is the first range measurment.

If the filter is initialized at some point other than during the first three cycles, the covariance matrix is initialized as above, but $\dot{R}$ and $\ddot{R}$ in the state vector are computed from first and second differences of range measurements.

The state vector $\underline{R}$ was used in the above discussion, but the formulation remains the same for azimuth and elevation with the following exception for the initialization of the covariance matrix:

$$\underline{P}(1/0) = \begin{bmatrix} 10S_A & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & .1 & 0 \\ 0 & 0 & 0 & .1 \end{bmatrix}$$

where S_A is replaced by S_E for the elevation covariance matrix.

4.5.2.3 Outstanding Features

Refer to subparagraph 4.5.1.3.

4.5.2.4 Limitations

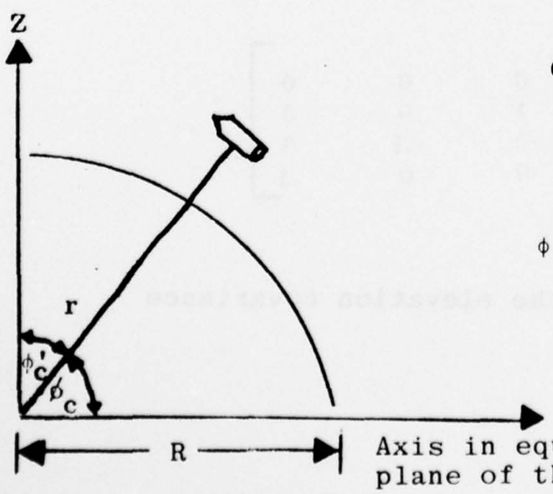
Refer to subparagraph 4.5.1.4.

4.5.3 Acceleration Due to Gravity

Assuming that the earth is a pear-shaped oblate spheroid, the gravity potential (Fisher Potential Model) can be expressed as a function of geocentric co-latitude, the complement of geocentric latitude and the radial distance from the center of the earth.

The gravity potential formula is:

$$V(r, \Phi'_c) = \frac{GM}{R} \cdot \left[\frac{R}{r} + \frac{JR^3}{r^3} \left(\cos^2 \Phi'_c - \frac{1}{3} \right) + \frac{DR^5}{35\, r^5} \left(35 \cos^4 \Phi'_c - 30 \cos^2 \Phi'_c + 3 \right) - H \frac{R^4}{r^4} \left(\cos^3 \Phi'_c - \frac{3}{5} \cos \Phi'_c \right) \right]$$



Axis in equatorial plane of the earth

GM Earth gravitation constant
J 1st oblateness correction
D 2d oblateness correction
H Pear shapness correction
r Geocentric radius
R Equatorial radius of the earth
ϕ'_c Geocentric Colatitude

Noting that, $\cos \phi_c^1 = Z/r$, we can write

$$V(r, Z) = \frac{GM}{R} \left[\frac{R}{r} \quad \frac{JR^3}{r^3} \left(\frac{Z^2}{r^2} \quad \frac{1}{3} \right) \right.$$

$$\left. + \frac{DR^5}{35r^5} \left(35 \frac{Z^4}{r^4} - 30 \frac{Z^2}{r^2} + 3 \right) - H \frac{R^4}{r^4} \left(\frac{Z^3}{r^3} - \frac{3}{5} \frac{Z}{r} \right) \right]$$

The gravity acceleration components can be obtained by taking the negative gradient of the potential V(r,z), that is,

$$r = f\ (X,\ Y,\ Z)$$

then $V(r,Z) = U[f(x,y,z)z]$

and

$$g_x = \frac{\partial U}{\partial x} \quad g_y = -\frac{\partial U}{\partial y} \quad g_z = -\frac{\partial U}{\partial z}$$

where,

$$\frac{\partial U}{\partial x} = \frac{\partial V}{\partial r} \cdot \frac{\partial f}{\partial x}$$

$$\frac{\partial U}{\partial y} = \frac{\partial V}{\partial r} \quad \frac{\partial f}{\partial y}$$

$$\frac{\partial U}{\partial z} = \frac{\partial V}{\partial r} \quad \frac{\partial f}{\partial z} + \frac{\partial V}{\partial z}$$

since,

$$r = f(x, y, z) = + (X^2 + Y^2 + Z^2)^{1/2}$$

then

$$\frac{\partial f}{\partial x} = \frac{x}{r}, \quad \frac{\partial f}{\partial y} = \frac{y}{r}, \quad \frac{\partial f}{\partial z} = \frac{z}{r}$$

and

$$\frac{\partial V}{\partial r} = -\frac{GM}{R_e}\left[\frac{R_e}{r^2} + J R_e^3\left(\frac{5Z^2}{r^6} - \frac{1}{r^4}\right)\right.$$

$$\left. + D R_e^5\left(\frac{9Z^4}{r^{10}} - \frac{6Z^2}{r^8} + \frac{3}{7r^6}\right) - H R_e^4\left(\frac{7Z^3}{r^8} - \frac{3Z}{r^6}\right)\right]$$

$$\frac{\partial V}{\partial z} = \frac{GM}{R_e}\left[\frac{2JR_e^3 Z}{r^5} + D R_e^5\left(\frac{4Z^3}{r^9} - \frac{12Z}{7r^7}\right) - H R_e^4\left(\frac{3Z^2}{r^7} - \frac{3}{5r^5}\right)\right]$$

4.5.4 Transformation Routines

4.5.4.1 Transformation from Radar Measurement to Rectangular Position Coordinates

From Figure 15, the transformation from radar measurements R, A and E to rectangular position coordinates u, v and w is

$$\left.\begin{aligned} u &= R \cos E \sin A \text{ -- East positive} \\ v &= R \cos E \cos A \text{ -- North Positive} \\ w &= R \sin E \text{ -- Up positive} \end{aligned}\right\} \text{Tangent plane radar oriented.} \qquad (32)$$

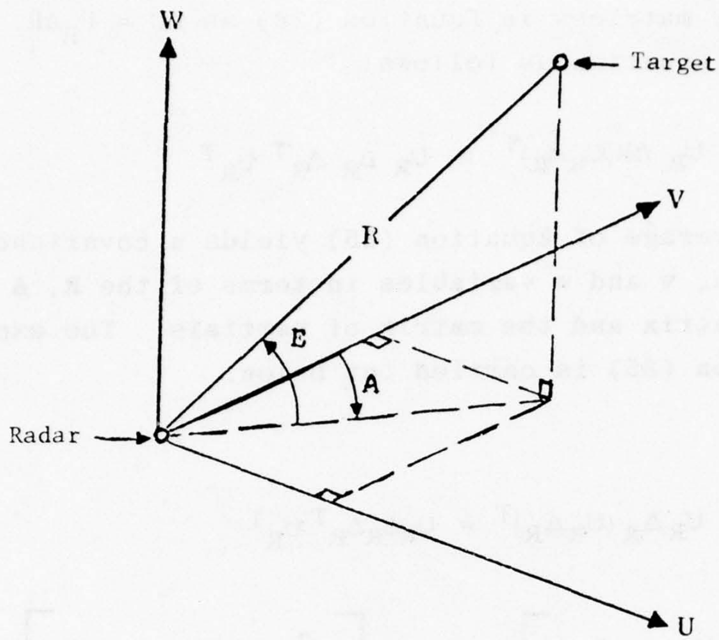


Figure 15. Radar Coordinate System.

A small change in any one of the variables, u for example, could be expressed in terms of a Taylor series expansion about the mean value $\bar{u}$ for small deviations as

$$\Delta u = \Delta R \frac{\partial u}{\partial R} \Big|_{\bar{R}} + \Delta A \frac{\partial u}{\partial A} \Big|_{\bar{A}} + \Delta E \frac{\partial u}{\partial E} \Big|_{\bar{E}} \tag{33}$$

when higher order terms are neglected. This expansion for all three variables u, v and w may be written in matrix form as

$$\begin{bmatrix} \Delta u \\ \Delta v \\ \Delta w \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial R} & \frac{\partial u}{\partial A} & \frac{\partial u}{\partial E} \\ \frac{\partial v}{\partial R} & \frac{\partial v}{\partial A} & \frac{\partial v}{\partial E} \\ \frac{\partial w}{\partial R} & \frac{\partial w}{\partial A} & \frac{\partial w}{\partial E} \end{bmatrix} \begin{bmatrix} \Delta R \\ \Delta A \\ \Delta E \end{bmatrix} \tag{34}$$

Designating the matrices in Equation (34) as $\Delta U = U_R \Delta R$, we can square the equation as follows:

$$\Delta u \, \Delta u^T = U_R \, \Delta R (U_R \Delta_R)^T = U_R \, \Delta_R \, \Delta_R^T \, U_R^T \tag{35}$$

The ensemble average of Equation (35) yields a covariance matrix of the u, v and w variables in terms of the R, A and E covariance matrix and the matrix of partials. The expansion of Equation (35) is carried out below.

Squaring

$$\Delta u \Delta u^T = U_R \Delta_R (U_R \Delta_R)^T = U_R \Delta_R \Delta_R^T U_R^T$$

$$\begin{bmatrix} \Delta u^2 & \Delta u \, \Delta v & \Delta u \, \Delta w \\ \Delta v \, \Delta u & \Delta v^2 & \Delta v \, \Delta w \\ \Delta w \, \Delta u & \Delta w \Delta v & \Delta w^2 \end{bmatrix} = U_R \begin{bmatrix} \Delta_R^{\ 2} & \Delta R \Delta E & \Delta R \Delta A \\ \Delta E \Delta R & \Delta E^2 & \Delta E \Delta A \\ \Delta A \Delta R & \Delta A \Delta E & \Delta A^2 \end{bmatrix} U_R^{\ T}$$

Taking the ensemble average

$$\begin{bmatrix} \sigma^2 u & \sigma uv & \sigma uw \\ \sigma vu & \sigma v^2 & \sigma vw \\ \sigma wu & \sigma wv & \sigma w^2 \end{bmatrix} = U_R \begin{bmatrix} \sigma_R^{\ 2} & \sigma_{RE} & \sigma_{RA} \\ \sigma_{ER} & \sigma_E^{\ 2} & \sigma_{EA} \\ \sigma_{AR} & \sigma_{AE} & \sigma_A^{\ 2} \end{bmatrix} U_R^T$$

$$= U_R \begin{bmatrix} \sigma_R^{\ 2} & 0 & 0 \\ 0 & \sigma_E^{\ 2} & 0 \\ 0 & 0 & \sigma_A^{\ 2} \end{bmatrix} U_R^T \quad \text{All cross correlations are zero.}$$

$$S_1 = U_R \, S_R \, U_R^T \tag{36}$$

Normally Equation (36) is written in the general terms

$$S_y = M_x S_x M_x^T \tag{37}$$

where

M_x is the matrix of partials.

This form will be used throughout the remainder of the report.

The elements of the matrix of partials M_x from Equation (32) are:

$$\left.\begin{aligned}
u_R &= \cos E \sin A \\
u_A &= R \cos E \cos A \\
u_E &= -R \sin E \sin A \\
v_R &= \cos E \cos A \\
v_A &= R \cos E \sin A \\
v_E &= -R \sin E \cos A \\
w_R &= \sin E \\
w_A &= 0 \\
w_E &= R \cos E
\end{aligned}\right\} \tag{38}$$

The errors in the u, v and w coordinates are

$$S_1 = \begin{bmatrix} \sigma_u{}^2 & \sigma uv & \sigma uw \\ \sigma uv & \sigma^2 v & \sigma vw \\ \sigma uw & \sigma vw & \sigma w^2 \end{bmatrix} = M_x \begin{bmatrix} \sigma_R^2 & 0 & 0 \\ 0 & \sigma_A{}^2 & 0 \\ 0 & 0 & \sigma_E^2 \end{bmatrix} M_x{}^T \qquad (39)$$

The covariances are:

$$S_1 = \begin{bmatrix} u_R & u_E & u_A \\ v_R & v_E & v_A \\ w_R & w_E & 0 \end{bmatrix} \begin{bmatrix} \sigma_R{}^2 & 0 & 0 \\ 0 & \sigma_E{}^2 & 0 \\ 0 & 0 & \sigma_A{}^2 \end{bmatrix} U_R{}^T$$

$$= \begin{bmatrix} u_R\sigma_R{}^2 & u_E\sigma_E{}^2 & u_A\sigma_A{}^2 \\ v_R\sigma_R{}^2 & v_E\sigma_E{}^2 & v_A\sigma_A{}^2 \\ w_R\sigma_R{}^2 & w_E\sigma_E{}^2 & 0 \end{bmatrix} \begin{bmatrix} u_R & v_R & w_R \\ u_E & v_E & w_E \\ u_A & v_A & 0 \end{bmatrix}$$

$$= \begin{bmatrix} u_R{}^2\sigma_R{}^2 + u_E{}^2\sigma_E{}^2 + u_A{}^2\sigma_A{}^2 & v_Ru_R\sigma_R{}^2 + v_Eu_E\sigma_E{}^2 + v_Au_A\sigma_A{}^2 & w_Ru_R\sigma_R{}^2 + w_Eu_E\sigma_E{}^2 \\ v_Ru_R\sigma_R{}^2 + v_Eu_E\sigma_E{}^2 + v_Au_A\sigma_A{}^2 & v_R{}^2\sigma_R{}^2 + v_E{}^2\sigma_E{}^2 + v_A{}^2\sigma_A{}^2 & w_Rv_R\sigma_R{}^2 + v_Ew_E\sigma_E{}^2 \\ w_Ru_R\sigma_R{}^2 + w_Eu_E\sigma_E{}^2 & w_Rv_R\sigma_R{}^2 + v_Ew_E\sigma_E{}^2 & w_R{}^2\sigma_R{}^2 + w_E{}^2\sigma_E{}^2 \end{bmatrix}$$

$$
\begin{aligned}
\sigma_u^2 &= u_R^2 \sigma_R^2 + u_E^2 \sigma_E^2 + u_A^2 \sigma_A^2 \\
\sigma_{uv} &= v_R u_R \sigma_R^2 + v_E u_E \sigma_E^2 + v_A u_A \sigma_A^2 \\
\sigma_{uw} &= w_R u_R \sigma_R^2 + w_E u_E \sigma_E^2 \\
\sigma_v^2 &= v_R^2 \sigma_R^2 + v_E^2 \sigma_E^2 + v_A^2 \sigma_A^2 \\
\sigma_{wv} &= w_R v_R \sigma_R^2 + v_E w_E \sigma_E^2 \\
\sigma_w^2 &= w_R^2 \sigma_R^2 + w_E^2 \sigma_E^2
\end{aligned}
\tag{40}
$$

These are the uncertainties in the calculated u, v and w set from the measured R, A and E coordinate set. Because the covariance matrix is symmetrical, σ_{vu}, σ_{wu} and σ_{wv} are not calculated.

4.5.4.2 Transformation of Position from Radar to Earth Centered Fixed Coordinates

The transformation of coordinates from u, v and w to x, y and z may be expressed as three rotations. The angles of rotation are shown in Figures 16 and 17. Performing the transformations, we readily obtain the following results:

1. First rotation (Figure 17a)

$$
\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{41}
$$

2. Second rotation (Figure 17b)

$$
\begin{bmatrix} U' \\ V' \\ W' \end{bmatrix} \begin{bmatrix} \cos\delta w & 0 & -\sin\delta w \\ 0 & 1 & 0 \\ \sin\delta w & 0 & \cos\delta w \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix} \tag{42}
$$

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3. Third rotation (Figure 17c)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & -\sin\theta_1 & \cos\theta_1 \\ 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \end{bmatrix} \begin{bmatrix} U' \\ V' \\ W' \end{bmatrix} \tag{43}$$

where $\theta_1 = \theta^* + \delta_n$. By substituting Equations (41) and (42) into Equation (43), the following results are obtained:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & -\sin\theta_1 & \cos\theta_1 \\ 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \end{bmatrix} \begin{bmatrix} \cos\delta w & 0 & -\sin\delta w \\ 0 & 1 & 0 \\ \sin\delta w & 0 & \cos\delta w \end{bmatrix} \begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{A} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{44}$$

or by multiplying the three matrices above; the result is shown in Equation (45).

$$\left.\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} C_1^1 & C_2^1 & C_3^1 \\ C_1^2 & C_2^2 & C_3^2 \\ C_1^3 & C_2^3 & C_3^3 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \right\} \tag{45}$$

where

$$\left.\begin{aligned}
C_1^1 &= \sin\theta_1 \sin a + \cos\theta_1 \sin\delta w \cos a \\
C_2^1 &= -\sin\theta_1 \cos a + \cos\theta_1 \sin\delta w \sin a \\
C_3^1 &= \cos\theta_1 \cos\delta w \\
C_1^2 &= \cos\delta w \cos a \\
C_2^2 &= \cos\delta w \sin a \\
C_3^2 &= -\sin\delta w \\
C_1^3 &= -\cos\theta_1 \sin a + \sin\theta_1 \sin\delta w \cos a \\
C_2^3 &= \cos\theta_1 \cos a + \sin\theta_1 \sin\delta w \sin a \\
C_3^3 &= \sin\theta_1 \cos\delta w
\end{aligned}\right\} \tag{46}$$

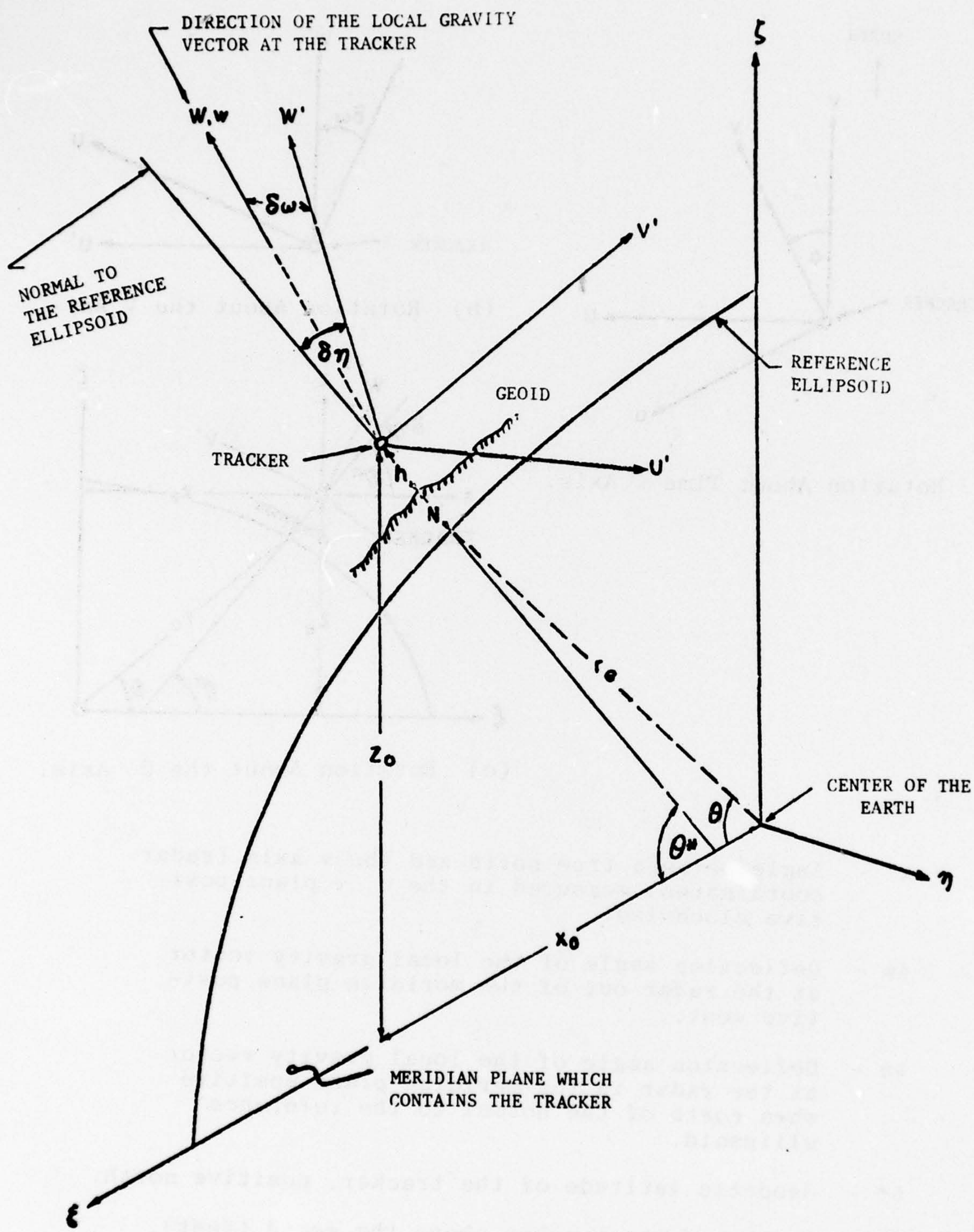
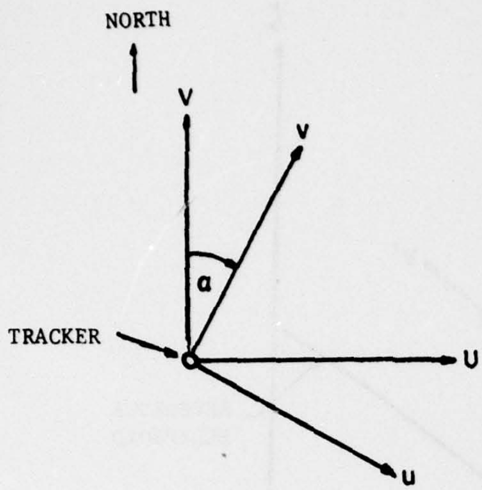
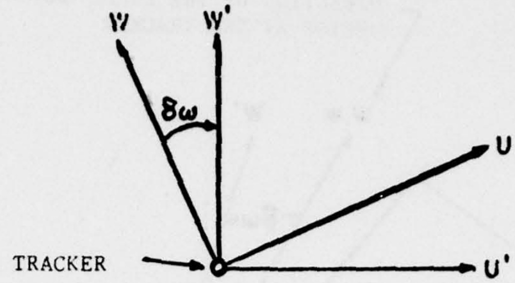


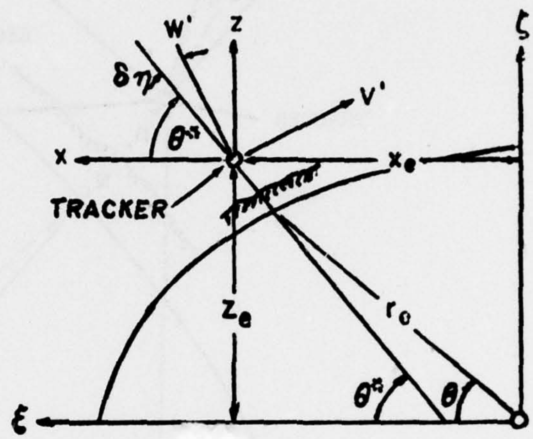
Figure 16. Definition of the Coordinate Systems.



(a) Rotation About Time ω Axis.



(b) Rotation About the V Axis.



(c) Rot[illegible] About the U' Axis.

α - Angle between true north and the v axis (radar coordinates) measured in the u, v plane positive clockwise.

δw - Deflection angle of the local gravity vector at the radar out of the meridian plane positive west.

δu - Deflection angle of the local gravity vector at the radar in the meridian plane positive when north of the normal to the reference ellipsoid.

θ* - Geodetic latitude of the tracker, positive north.

h - Height of the tracker above the geoid (feet).

N - Geoidal separation-positive when the geoid is above the reference ellipsoid.

Figure 17. Sequence of Rotations Used to Derive the Position Transformation Equations.

Each rotation matrix in Equation (44) is orthogonal, and for this reason, the inverse of Equation (45) is simply:

$$\left.\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \underbrace{\begin{bmatrix} C_1^1 & C_1^2 & C_1^3 \\ C_2^1 & C_2^2 & C_2^3 \\ C_3^1 & C_3^2 & C_3^3 \end{bmatrix}}_{A^{-1} = A^T} \begin{bmatrix} x \\ y \\ z \end{bmatrix}\right\} \quad (47)$$

The transformation of position from x, y and z to earth centered fixed coordinates ξ, η and ζ is then simply the following translation (see Figures 16 and 17a):

$$\left.\begin{aligned} \xi &= (x + x_e) \\ \eta &= (y) \\ \zeta &= (z + z_e) \end{aligned}\right\} \quad (48)$$

where

$$\left.\begin{aligned} x_e &= r_e \cos\theta + (h + N)\cos\theta^* \\ z_e &= r_e \sin\theta + (h + N)\sin\theta^* \end{aligned}\right\} \quad (49)$$

r_e is the radius of the reference ellipsoid at the latitude of the tracker, which is given by the following equation:

$$r_e = \frac{a\ (1-e)}{\sqrt{1 - (2e - e^2)\cos^2\theta}} \quad (50)$$

The relation between geodetic latitude θ* and geocentric latitude θ may be easily found from Equation (49). This is as follows:

$$\tan\theta = (1 - e)^2 \tan\theta^* \tag{51}$$

Writing Equation (48) in more convenient form, we get

$$\left.\begin{aligned} \xi &= x + C_1^{12} \\ \eta &= y \\ \zeta &= z + C_1^{12} \end{aligned}\right\} \tag{52}$$

$$\left.\begin{aligned} C_1^{12} &= r_e \cos\theta + (h + N)\cos\theta^* \\ C_2^{12} &= r_e \sin\theta + (h + N)\sin\theta^* \end{aligned}\right\} \tag{53}$$

$$r_e = \frac{a\,(1 - e)}{\sqrt{1 - (2e - e^2)\cos^2\theta}}$$

$$\theta = \tan^{-1}\left[(1 - e)^2 \tan\theta^*\right]$$

We will use values of a and e as determined by the Fisher Ellipsoid (1960)

a = 20,925,741 ft

e = $3.35233051 \times 10^{-3}$

e is ellipticity

4.5.4.3 Error Propagation

Using the equation for propagation of variance

$$S_y = M_x S_x M_x^T$$

and operating on Equation (45) to find the matrix of partials M_x one finds that M_x equals the matrix of C_ns.

$$M_x = \begin{bmatrix} C_1^1 & C_2^1 & C_3^1 \\ C_1^2 & C_2^2 & C_3^2 \\ C_1^3 & C_2^3 & C_3^3 \end{bmatrix} = A \tag{54}$$

Then the covariance matrix for the x, y and z coordinates is

$$S_2 = AS_1A^T \tag{55}$$

where S_1 is the covariance matrix in the radar pad coordinates U, V and W.

4.6 WHITE SANDS MISSILE RANGE

4.6.1 The QD Digital Filter

4.6.1.1 In early 1965, due to expanding requirements for the range real-time digital computer, the need for a fast, efficient, and storage-conservant data filter became apparent. Mr. W. A. McCool, Director of the Analysis and Computation Directorate, having an extensive background in electronic engineering and analog computing developed an idea and procedure whereby techniques of a basic analog feedback filter could be applied to digital data. The development of concepts of real-time data filtering at WSMR began with this original idea, and the credit, although others have contributed significantly to usage development, belongs to Mr. McCool.

The first operational filter developed at WSMR under this theory was the Digital Filter Experimental (DFX), and it was used operationally as late as December 1969. DFX was a fast (less than 1 millisecond of 7044 time per component at a 9-point span) and efficient (performed equally as well as constrained least squares in terms of smoothness and was more conservant of computing time at equivalent point spans) data filter used for all types of range safety applications. DFX was employed as a variable span filter with changes in span occurring according to observed acceleration in the filtered data. The computing time required for DFX was directly proportional to the length of the filter span and there were certain elements, in array form, which required retention in computer core storage.

Many ideas innovated in DFX carried through in the enhancement of QD. The conversion to QD was implemented because of the following advantages: (1) filter computing time for QD is the same regardless of span (and faster than DFX at a minimum equivalent span); (2) core storage requirements are about 2/3 that of DFX; and (3) QD performance over data containing random noise is decidely better than that of DFX.

The QD digital filter was developed at WSMR in 1967, and it has since been used in computer programs for real-time mission support and in a variety of other types of deferred programs. The basic concept of the QD filter is not new. For example, the alpha-beta (α-β) filter was developed more than ten years ago for target position prediction in track-while-scan radars. The significant aspects of the QD filter lie in (1) minimal computing time, (2) unique mathematical formulation based on an nth order polynomial

curve fit and (3) extension to digital data smoothing. The purpose of the next section is to summarize the development of the QD formulas for the second and third order fits.

4.6.1.2 QD Theory

Given a set of discrete data values with equal increments of the argument,

$$x(t_n) = x_0, x_1, x_2, \ldots\ldots\ldots\ldots x_N, \tag{1}$$

$$h = t_{n+1} - t_n, \tag{2}$$

the second order QD formulas for sequential filtering these data are described as follows:

QD is a recursive predict-correct procedure. The prediction formulas are:

$$\bar{\ddot{x}}_{n+1} = \hat{\ddot{x}}_n, \tag{3}$$

where:

$\hat{\ddot{x}}_n$ - second derivative estimate from preceding step,

$\bar{\ddot{x}}_{n+1}$ - predicted second derivative;

$$\bar{\dot{x}}_{n+1} = \hat{\dot{x}}_n + h\hat{\ddot{x}}_n, \tag{4}$$

$\bar{\dot{x}}_{n+1}$ - predicted first derivative,

$\hat{\dot{x}}_n$ - first derivative estimate from preceding step;

$$\bar{x}_{n+1} = \hat{x}_n + h\hat{\dot{x}}_n + \frac{h^2}{2}\hat{\ddot{x}}_n, \tag{5}$$

or

$$\bar{x}_{n+1} = \hat{x}_n + \frac{h}{2}(\bar{\dot{x}}_{n+1} + \hat{\dot{x}}_n), \tag{6}$$

where

$\bar{x}_{n+1}$ - predicted data value,

$\hat{x}_n$ - data value estimate from preceding step.

The prediction formulas (4) and (5) are Taylor series truncated for the second degree curve fit. Formula (6) is a more computationally efficient version of formula (4) obtained by substituting formula (4) into formula (5). Note that formula (6) is the familiar trapezoidal integration formula.

The corrections of the predicted values $\bar{\ddot{x}}_{n+1}$, $\bar{\dot{x}}_{n+1}$ and $\bar{x}_{n+1}$ are based upon the difference between the given and predicted data values, i.e.,

$$\Delta x = x_{n+1} - \bar{x}_{n+1} \tag{7}$$

The correction formulas are:

$$\hat{\ddot{x}}_{n+1} = \bar{\ddot{x}}_{n+1} + k_1 \Delta x \tag{8}$$

$$\hat{\dot{x}}_{n+1} = \bar{\dot{x}}_{n+1} + k_2 \Delta x \tag{9}$$

$$\hat{x}_{n+1} = \bar{x}_{n+1} + k_3 \Delta x \tag{10}$$

where

$\hat{x}_{n+1}$ - current estimate of the true data value

$\hat{\dot{x}}_{n+1}$ - current estimate of the corresponding first derivative

$\hat{\ddot{x}}_{n+1}$ - current estimate of the corresponding second derivative

k_1, k_2, k_3 - correction coefficients which minimize the error of the estimates

Formulas (4), (6), (7), (8), (9) and (10) prescribe the computational procedure of the second order QD digital filter. The QD formulas use the estimates evaluated in the preceding step and are, therefore, said to be recursive. This being the case, QD must be initialized, i.e., initial estimates must be obtained independently. The most common procedure is to compute the initial estimates from the first few data values with a conventional least-squares curve fit.

The QD formulas were derived from the constrained least-squares (CLS) filter applied to M given data values with estimates determined for the current given data value, i.e., m=M. In the CLS filter, intercept and slope constraints are applied at the oldest value end of the M value span, i.e., m=0. This means that the polynomial fit to the M data values in the least-squares sense must also contain the estimates of the true value and the corresponding first derivative at the span point m=0. Using these constraints in the second order CLS filter, the predicted data value and its derivative can be obtained with the truncated Taylor series.

$$\bar{\dot{x}}_{n+1} = \hat{\dot{x}}_{n-M+1} + M h \hat{\ddot{x}}_{n-M+1}, \tag{11}$$

$$\bar{x}_{n+1} = \hat{x}_{n-M+1} + M h \hat{\dot{x}}_{n-M+1} + \frac{(MH)^2}{2} \hat{\ddot{x}}_{n-M+1} \tag{12}$$

Where the subscript n-M+1 corresponds to m=0 in the filter span and $\hat{x}_{n-M+1}$ and $\hat{\dot{x}}_{n-M+1}$ are the constraints which were computed in the preceding step as $\hat{x}_{n-M+2}$ and $\hat{\dot{x}}_{n-M+2}$ using

Taylor series again. Since $\hat{x}_{n-M+1}$ and $\hat{\dot{x}}_{n-M+1}$ are fixed, only $\hat{\ddot{x}}_{n-M+1}$ can be adjusted to make formula (12) fit the M data values in the least-squares sense. Let $\Delta\ddot{x}$ be the required correction. Then

$$\hat{\ddot{x}}_{n+1} = \hat{\ddot{x}}_{n-M+1} + \Delta\ddot{x}. \tag{13}$$

We note here that the second derivative is constant across the second order CLS filter span. Applying the correction to formulas (11) and (12) gives the estimates,

$$\hat{\dot{x}}_{n+1} = \hat{\dot{x}}_{n-M+1} + M\,h\,(\hat{\ddot{x}}_{n-M+1} + \Delta\ddot{x}), \tag{14}$$

$$\hat{x}_{n+1} = \hat{x}_{n-M+1} + M\,h\,\hat{\dot{x}}_{n-M+1} + \frac{(Mh)^2}{2}(\ddot{x}_{n-M+1} + \Delta\ddot{x}). \tag{15}$$

Subtracting formulas (11) and (12) from formulas (14) and (15), respectively, gives

$$\hat{\dot{x}}_{n+1} = \bar{\dot{x}}_{n+1} + M\,h\,\Delta\ddot{x}, \tag{16}$$

$$\hat{x}_{n+1} = \bar{x}_{n+1} + \frac{(Mh)^2}{2}\,\Delta\ddot{x} \tag{17}$$

Comparison of formulas (13), (16) and (17) with formulas (8), (9) and (10) reveals the functional relationship among the correction coefficients k_1, k_2 and k_3. From formulas (13) and (8) we have

$$\Delta\ddot{x} = k_1 \Delta x. \tag{18}$$

Then from formulas (16), (9) and (18),

$$k_2 = M\,h\,k_1, \tag{19}$$

and from formulas (17), (10) and (18),

$$k_3 = \frac{(Mh)^2}{2} k_1. \tag{20}$$

In the QD theory k_1 is developed as a function of the corresponding CLS filter span M, i.e.,

$$k_1 = k_1(M). \tag{21}$$

Thus, when M is arbitrarily specified, the QD correction coefficients are determined with formulas (21), (19) and (20).

We note that QD prediction formulas (4) and (5) do not contain the constraints $\hat{x}_{n-M+1}$ and $\hat{\dot{x}}_{n-M+1}$ which are eliminated in the QD theory. This is easily shown as follows. Recognizing that any two points contained in the fitted curve of the preceding step can be related by a Taylor series, we can write

$$\hat{\dot{x}}_n = \hat{\dot{x}}_{n-M+1} + (M-1)h\,\hat{\ddot{x}}_{n-M+1} \tag{22}$$

$$\hat{x}_n = \hat{x}_{n-M+1} + (M-1)h\,\hat{\dot{x}}_{n-M+1} + \frac{(M-1)^2h^2}{2}\,\hat{\ddot{x}}_{n-M+1} \tag{23}$$

QD prediction formulas (4) and (5) are obtained by combining formulas (22), (23), (11) and (12) to eliminate $\hat{x}_{n-M+1}$ and $\hat{\dot{x}}_{n-M+1}$. Thus, it is now clear that the second order fitted curve established in the preceding step and containing the intercept and slope constraints is used for the prediction phase of the current computing step, i.e., formulas (3), (4) and (6).

The computing formulas of the third order QD digital filter are as follows:

$$\bar{\ddot{x}}_{n+1} = \hat{\ddot{x}}_n + h\,\hat{\dddot{x}}_n \qquad (24)$$

$$\bar{\dot{x}}_{n+1} = \hat{\dot{x}}_n + \frac{h}{2}\left(\bar{\ddot{x}}_{n+1} + \hat{\ddot{x}}_n\right) \qquad (25)$$

$$\bar{x}_{n+1} = \hat{x}_n + \frac{h}{2}\left(\bar{\dot{x}}_{n+1} + \hat{\dot{x}}_n\right) + \frac{h^3}{12}\hat{\dddot{x}}_n \qquad (26)$$

$$\Delta x = x_{n+1} - \bar{x}_{n+1} \qquad (27)$$

$$\hat{\dddot{x}}_{n+1} = \hat{\dddot{x}}_n + k_1\Delta x \qquad (28)$$

$$\hat{\ddot{x}}_{n+1} = \hat{\ddot{x}}_n + k_2\Delta x \qquad (29)$$

$$\hat{\dot{x}}_{n+1} = \hat{\dot{x}}_n + k_3\Delta x \qquad (30)$$

$$\hat{x}_{n+1} = \hat{x}_n + k_4\Delta x \qquad (31)$$

In the third order QD formulation the intercept and slope constraints are applied and the second and third derivatives are adjusted to satisfy the least-squares error criterion. The correction coefficients k_1, k_2, k_3 and k_4 are developed as functions of M, the equivalent third order CLS filter span. Note that formulas (25) and (26) have been modified for computing efficiency.

In its basic formulation, QD is a real-time filter, i.e., the argument of the estimates corresponds to the latest value accepted by the filter. On the other hand, estimates can be obtained for the data value at the oldest end of the span which are significantly better than the corresponding real-time estimates. Such estimates, which are often called "smoothed" data, correspond to the constraints computed in the CLS filter. Smoothed estimates are simply obtained with a Taylor series expanded about the real-time estimates, i.e.,

$$\widehat{\hat{\dot{x}}}_{n+1} = \hat{\dot{x}}_{n+1} - (M-1)h\,\hat{\ddot{x}}_{n+1} \tag{32}$$

$$\widehat{\hat{x}}_{n+1} = \hat{x}_{n+1} - \frac{(M-1)}{2}h(\widehat{\hat{\dot{x}}}_{n+1} + \hat{\dot{x}}_{n+1}) \tag{33}$$

for the second order QD filter and

$$\widehat{\hat{\ddot{x}}}_{n+1} = \hat{\ddot{x}}_{n+1} - (M-1)\,h\,\widehat{\dddot{x}}_{n+1} \tag{34}$$

$$\widehat{\hat{\dot{x}}}_{n+1} = \hat{\dot{x}}_{n+1} - \frac{(M-1)h}{2}\widehat{\hat{\ddot{x}}}_{n+1} + \hat{\ddot{x}}_{n+1}) \tag{35}$$

$$\widehat{\hat{x}}_{n+1} = \hat{x}_{n+1} - \frac{(M-1)h}{2}(\widehat{\hat{\dot{x}}}_{n+1} + \hat{\dot{x}}_{n+1}) \qquad \frac{(M-1)^3h^3}{12}\dddot{x}_{n+1} \tag{36}$$

for the third order QD filter.

The basic QD filter, used for WSMR safety applications, operates at variable point spans which are adjusted according to acceleration. The filter spans vary from as few as 17 to as many as 46 points. When QD is used to filter FPS-16 data, 2 raw data points are averaged to form 1 filter input point. The span of the filter is varied according to a philosophy that the higher the acceleration the shorter the span. Thus, when accelerations are high, maximum filter response to changes in the input curve are achieved; when accelerations are low, the benefits of maximum smoothing are available.

The filter is always initialized or re-initialized at the minimum span. Should span adjustment be deemed necessary, the span is lengthened or shortened by only 1 point per filter cycle. A table of acceleration values determines what span the filter should be or whether the span should be increasing or decreasing. This table of values can be formulated according to the performance of a particular vehicle to both minimize "overshoot" and achieve maximum smoothing as necessary.

4.6.1.3 Outstanding Features

Several features presented here will make WSMR QD filter operational use more clear and will explain why it is extensively applied.

a. QD Spike Filter. QD has an inherent spike filter, not previously discussed, which vastly improves filter operation. The recursive nature of the filter is employed such that after the prediction equation is applied and ΔX is computed (ΔX = input position minus predicted input position), the magnitude of ΔX is compared to a predetermined spike limit L. L is an amount of position change reflecting an acceleration that the particular vehicle under observation is not capable of achieving. L must be formulated such that values of ΔX equal to or exceeding L can be considered to be noise or spikes in the raw data.

When a value of ΔX is observed to equal or exceed L, a spike counter (C) is incremented and the predicted value of position ($\bar{X}n + 1$) is used for filter input rather than the raw input position ($Xn + 1$). This substitution is allowed to occur n/2 times for a given filter span (n). If n/2 + 1 spikes are observed on a filter cycle, re-initialization is automatically forced. In general usage at WSMR, a double spike limit is employed. If $|\Delta X| > L1$ and $C1 \geq n/2 + 1$, then the same test is made for an L2 and C2. The C1 and C2 counters are incremented and decremented according to the magnitude of ΔX. For example, if the value of $\Delta X > L1$ but $< L2$, only C1 is incremented. This double spike limit allows a fine degree of spike editing using L1 with L2 available to delete larger spikes and avoid unnecessary re-initialization.

b. Initialization. A 5 point least-squares curve fit is used for initialization of the recursive QD filter; the 5 point initialization array is shifted, and the

latest raw data value is inserted on each cycle of the filter. This can enable the filter to be initialized in less than $5\Delta t$ (Δt being the data sampling interval). In the case of meteorological use of the QD filter for computation of wind velocity, a 7 point least-squares curve fit is used for both initialization and to form the input to the filter on each cycle. This is opposed to using the raw input data for filter input and, when used with the midpoint smoothing technique explained in subparagraph 4.6.1.2, provides optimum smooth data which lags real time by approximately 2 seconds. Studies are currently underway to modify the meteorological QD for use as a real-time filter for filtering rough singular measurements. This would be advantageous for forming multiple station solutions from some instrumentation previously described.

c. Variable Filter Span. Variable filter spans allow more general use of the filter through the increased versatility gained by automatically adapting the filter to changes in observed acceleration.

d. Filter Speed and Storage Requirements. QD is a real-time data filter that precisely meets the design and development goals. The minimum storage requirements and computing speed of QD must be rated as outstanding features.

4.6.1.4 Limitations

Since other ranges would undoubtedly have to adjust the QD filter to satisfy their particular needs, such adjustments could be classed as limitations.

The only limitation or possible problem area at WSMR with QD is that extremely rough input data, which

passes through the spike filter, will be interpreted as an acceleration change and will cause undesired shortening of the filter span. While this is not a major problem at WSMR, manual overrides have been programmed into the WSMR QD filter for fixing the filter at a particular span length and for providing different values of the spike limits. This was done primarily for orbital support operations and aircraft tracking, rather than range safety during missile flight experiments.

4.7 ARMAMENT DEVELOPMENT AND TEST CENTER

4.7.1 Least-Squares Polynomial Moving Arc Filter Using Recursive Sums

The real-time range safety filter in current use at the ADTC employs a modified recursive scheme for computing the sums required to perform least-squares polynomial moving arc smoothing.

The matrix equation for this type of smoothing may be written as:

$$A = C^{-1}B$$

where A is the vector of polynomial coefficients, C is the normal matrix with elements

$$C_{ij} = \sum_{m=1}^{n} t_m^{\,i+j-2} \qquad \begin{array}{l} i = 1,2,\ldots,\ d+1 \\ j = 1,2,\ldots,\ d+1 \end{array}$$

and B is a vector whose kth element is of the form

$$b_k = \sum_{i=1}^{n} y_i t_i^{\,k} \qquad \text{for } k = 0,1,2,\ldots,d,$$

d being the degree of the polynomial; t_i, the time of the sample referenced to the midpoint of the span under consideration; and y_i, the coördinate (x,y or z) at time t_i.

For instance, if one has obtained:

$$b_i(1) = \sum_{k=1}^{n} y_k t_k^{i-1}, \quad t_k = T_k - T_{\frac{n-1}{2}}$$

and wishes to find:

$$b_i(2) = \sum_{k=2}^{n+1} y_k (t_k - \Delta)^{i-1}, \quad \Delta = T_{\frac{n+3}{2}} - T_{\frac{n+1}{2}},$$

one may do so in terms of $b_i^{(1)}$ by means of the recursion:

$$b_i(2) = \sum_{j=0}^{i-1} (-\Delta)^j \binom{i-1}{j} b_{i-j}^{(1)} + y_{n+1} \sum_{j=0}^{i-1} (-\Delta)^j \binom{i-1}{j} t_{n+1}^{i-j-1}$$

$$- y_1 \sum_{j=0}^{i-1} (-\Delta)^j \binom{i-1}{j} t_1^{i-j-1}$$

The elements of C are computed similarly by considering the $y_k=1$. However, if the data are evenly spaced, C is uniquely determined when the degree of the polynomial and the number of points smoothing has been selected, and C^{-1} can be pre-computed and stored for use in the smoothing computations. In this case, $\Delta=1$ and the equations for recursion are simplified. With this formulation no lengthy summations are required, and the number of operations, once initiated, are independent of the number of points used in the smoothing. Furthermore, by computing recursively $\Sigma\, y_k^2$, one can obtain the sum square

residuals of each span as an estimate of the error in the data by

$$S = \Sigma\, y_k^2 - A^T B$$

4.7.1.1 Features. The ADTC filter algorithm is recursive in the sense that it uses the output of a previous fixed time span as input to the next time span. Data at the newest time point is used to modify the preceding estimate, and the contribution of data at the oldest point in the preceding span is deleted from the estimate. This method, once the filter is initiated, allows for rapid calculation and minimal computer core requirements, and leads to a simple estimate of individual sensor data quality for use in source selection.

4.7.1.2 Limitations. The algorithm is unstable in the sense that it is unable to eliminate the effect of any past erroneous data on the recursive estimations. Filter initialization can be lengthy. For initialization of a 51-point filter, at a data sample rate of 10 samples per second, 5 seconds is required. The filter is most efficient for track of objects with smooth trajectories and is not adaptive to those performing intricate maneuvers.

4.7.2 Planned Filter Applications

The existing ADTC filter model was initially developed for a post-flight data reduction application. When used in this way, lengthy initialization times and lack of adaptiveness and stability were not of serious consequence. The adaptation of the filter to real time did not overcome these problems. As a result, the present filter is most effective on smooth flight profiles with radar data of good tracking quality. However, it is common for the tactical

type of vehicles tracked by ADTC radars to perform intricate maneuvers at low elevation tracking angles. The tracking of more than one vehicle during a mission is also common as is the tracking of rapidly accelerating air-to-air and air-to-ground missiles launched from maneuvering aircraft. The ADTC is presently in the midst of developing a centralized CRT oriented control center capability to replace existing decentralized plotboard facilities. Effective use of the new concepts and technology require that the existing computer software, algorithms and procedures be totally revamped. To adequately support these situations, a filter for specific ADTC application has been designed. (References 15 and 16.)

4.7.2.1 Filter Method. The replacement for the current ADTC real-time filter utilized a multiple exponential filter technique for each data channel. The design application has the following features:

a. Accepts data from multiple radars.

b. Filter weights may be selected to give zero, first or second degree polynomial filtering with QD, Kalman or no constraints imposed.

c. Three sets of filter weights may be applied to each channel of input.

d. A zero degree polynomial may be used to estimate noise variance.

e. Adaptation is accomplished through inter-filter velocity comparisons and residual trend detection.

f. Initialization is accomplished by means of an expanding memory least-squares polynomial algorithm.

The filter scheme makes use of the fact that several exponential filters can be applied in less computer time than is required for the present filter. Each data channel, then, is assigned three different exponential filters representing different effective memories or bandwidths. The calculations are performed for each filter at each data point. The optimum filter for the prevailing conditions of data quality, vehicle maneuvering and radar performance is selected through inter-filter comparisons for that channel. This enables the filters to adapt to prevailing conditions to avoid long settling periods after abrupt maneuvers, missile separations and data dropouts.

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